Department of Mathematics/Faculty of Nuclear Sciences and Physical Engineering	
Name and Surname of Candidate	Passport Number

Entrance examination in mathematics

example

Mathematical Engineering

1 (6 points) Solve the following differential equation

$$y'' - 2y' + 10y = 27x\mathrm{e}^x$$

together with conditions $y(0) = -2$ a $y'(0) = 1$.

2 (2 points) Calculate Wronskian (Wronski Determinant) for functions $\mathrm{e}^{-2x}, x\mathrm{e}^{-2x}, x^2\mathrm{e}^{-2x}$. Are these functions linearly dependent or not? Explain.

3 (5 points) Create a Taylor's series of the function $g(x) = \mathrm{e}^{3x}$ centered to the point $x = 0$. For which x the Taylor's series is convergent? Use the result to determine the sum of the following series:

$$\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{4^k k!}.$$

4 (10 points) Let

$$A = \left\{(x,y) \in \mathbb{R}^2 : \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^4 \leqslant \frac{2xy}{ab}\right\}.$$

be a two-dimensional area and a, b be positive parameters. By means of the mapping $x = ar\cos(\varphi)$, $y = br\sin(\varphi)$, calculate the integral $\int_A x^2y^2 \, \mathrm{d}(x,y)$.

5 (2 points) Find sum and product of all eigenvalues of the matrix $\mathbb{H} = \begin{pmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$.

6 (5 points) For which x the series $\sum_{n=1}^{\infty}\left(\frac{2^n}{n} + \frac{3^n}{n^2}\right)x^n$ is convergent?

7 (4 points) Let X be a linear normed space and $f : X \to \mathcal{R}$ a mapping given by $f(x) = \|x\|,\ x \in X$. Prove its continuity.

8 (6 points) Solve the differential equation

$$2xy - 2x + (x^2 + 3)y' = 0$$

with the initial condition $y(1) = 2$.

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Department of Mathematics/Faculty of Nuclear Sciences and Physical Engineering	
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Entrance examination in mathematics

example

Applied Algebra and Analysis

1 (6 points) Solve the following differential equation

$$y'' - 2y' + 10y = 27x\mathrm{e}^x$$

together with conditions $y(0) = -2$ a $y'(0) = 1$.

2 (2 points) Calculate Wronskian (Wronski Determinant) for functions $\mathrm{e}^{-2x}, x\mathrm{e}^{-2x}, x^2\mathrm{e}^{-2x}$. Are these functions linearly dependent or not? Explain.

3 (5 points) Create a Taylor's series of the function $g(x) = \mathrm{e}^{3x}$ centered to the point $x = 0$. For which x the Taylor's series is convergent? Use the result to determine the sum of the following series:

$$\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{4^k k!}.$$

4 (10 points) Let

$$A = \left\{(x,y) \in \mathbb{R}^2 : \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^4 \leqslant \frac{2xy}{ab}\right\}.$$

be a two-dimensional area and a, b be positive parameters. By means of the mapping $x = ar\cos(\varphi)$, $y = br\sin(\varphi)$, calculate the integral $\int_A x^2y^2 \,\mathrm{d}(x,y)$.

5 (2 points) Find sum and product of all eigenvalues of the matrix $\mathbb{H} = \begin{pmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$.

6 (5 points) For which x the series $\sum_{n=1}^{\infty}\left(\frac{2^n}{n} + \frac{3^n}{n^2}\right)x^n$ is convergent?

7 (5 points) Find Fourier transformation of function

$$f(x) = \begin{cases} x-2, & 2 \le x \le 3, \\ 4-x, & 3 \le x \le 4, \\ 0 & x < 2 \text{ nebo } x > 4. \end{cases}$$

8 (5 points) Find a solution of heat equation problem:

$$u_t = u_{xx}, \quad 0 < x < 1, \ t > 0$$
$$u(0,t) = u(1,t) = 0,$$
$$u(x,0) = x^2 - 1.$$

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Name and Surname of Candidate	Passport Number

Entrance examination in mathematics

example

Mathematical informatics

1 (6 points) Solve the following differential equation

$$y'' - 2y' + 10y = 27x\mathrm{e}^{x}$$

together with conditions $y(0) = -2$ a $y'(0) = 1$.

2 (2 points) Calculate Wronskian (Wronski Determinant) for functions $\mathrm{e}^{-2x}, x\mathrm{e}^{-2x}, x^2\mathrm{e}^{-2x}$. Are these functions linearly dependent or not? Explain.

3 (5 points) Create a Taylor's series of the function $g(x) = \mathrm{e}^{3x}$ centered to the point $x = 0$. For which x the Taylor's series is convergent? Use the result to determine the sum of the following series:

$$\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{4^k k!}.$$

4 (10 points) Let

$$A = \left\{(x, y) \in \mathbb{R}^2 : \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^4 \leqslant \frac{2xy}{ab}\right\}.$$

be a two-dimensional area and a, b be positive parameters. By means of the mapping $x = ar\cos(\varphi)$, $y = br\sin(\varphi)$, calculate the integral $\int_A x^2y^2 \,\mathrm{d}(x, y)$.

5 (2 points) Find sum and product of all eigenvalues of the matrix $\mathbb{H} = \begin{pmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$.

6 (5 points) For which x the series $\sum_{n=1}^{\infty}\left(\frac{2^n}{n} + \frac{3^n}{n^2}\right)x^n$ is convergent?

7 (5 points) Let $G = \left\{\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) : a, b, c, d \in \mathbb{Z} \text{ a } \det\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) = 1\right\}$. Show that G equipped with the standard matrix multiplication is a group. What can be said about the eigenvalues of a matrix $M \in G$ if the order of M in G is equal to a given positive integer k. Find in G (if it exists) an element of order 2, an element of order 4, and an element of order $+\infty$.

8 (5 points) Given the ring $\mathbb{Z}[i] := \{a + ib : a, b \in \mathbb{Z}\}$, where i is the imaginary unit and the operations + and $\times$ are defined as in the field $\mathbb{C}$ of complex numbers. Denote $\beta = i - 1 \in \mathbb{Z}[i]$. We say that $x \in \mathbb{Z}[i]$ is related to $y \in \mathbb{Z}[i]$ and write $x \sim y$, if there exists $w \in \mathbb{Z}[i]$ such that $x - y = \beta w$. Show that $\sim$ is an equivalence relation on $\mathbb{Z}[i]$. Decide whether $2i \sim 2 + i$.

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Entrance examination in mathematics

example

Applied Mathematical Stochastic Methods

1 (6 points) Solve the following differential equation

$$y'' - 2y' + 10y = 27x\mathrm{e}^{x}$$

together with conditions $y(0) = -2$ a $y'(0) = 1$.

2 (2 points) Calculate Wronskian (Wronski Determinant) for functions $\mathrm{e}^{-2x}, x\mathrm{e}^{-2x}, x^2\mathrm{e}^{-2x}$. Are these functions linearly dependent or not? Explain.

3 (5 points) Create a Taylor's series of the function $g(x) = \mathrm{e}^{3x}$ centered to the point $x = 0$. For which x the Taylor's series is convergent? Use the result to determine the sum of the following series:

$$\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{4^k k!}.$$

4 (10 points) Let

$$A = \left\{(x, y) \in \mathbb{R}^2 : \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^4 \leqslant \frac{2xy}{ab}\right\}.$$

be a two-dimensional area and a, b be positive parameters. By means of the mapping $x = ar\cos(\varphi)$, $y = br\sin(\varphi)$, calculate the integral $\int_A x^2y^2\,\mathrm{d}(x, y)$.

5 (2 points) Find sum and product of all eigenvalues of the matrix $\mathbb{H} = \begin{pmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$.

6 (5 points) For which x the series $\sum_{n=1}^{\infty}\left(\frac{2^n}{n} + \frac{3^n}{n^2}\right)x^n$ is convergent?

7 (5 points) Calculate the standard deviation of random variable described by probability density $g(x) = 16\Theta(x)x\mathrm{e}^{-4x}$, where $\Theta(x)$ is Heaviside unit-step function

$$\Theta(x) = \begin{cases} 0 & x \leqslant 0; \\ 1 & x > 0. \end{cases}$$

8 (5 points) Find a probability density function of two independent and identically distributed random variables $\mathcal{X}, \mathcal{Y}$ so that the sum $\mathcal{X} + \mathcal{Y}$ is exponentially distributed via probability density function $4\Theta(x)\mathrm{e}^{-4x}$. Advice: Use Laplace transform.

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