Jan Pabím

① Řešte + I s poč. podmínkou $y(1)=\sqrt[3]{2}$

$$(3x^4-27x^2+2x)(y^3-1)+3y^2(x^2-9)y'=0$$

separovatelná rce

$$\Big/\ \frac{1}{(y^3-1)(\cancel{3x^4-27x^2+2x})(x^2-9)}$$

$y\neq 1,\ x\neq \pm 3$

$$y'\frac{3y^2}{(y^3-1)}+\frac{(3x^4-27x^2+2x)}{(x^2-9)}=0$$

$$\underbrace{\int\frac{3y^2\,dy}{(y^3-1)}}_{\bullet}+\underbrace{\int\frac{(3x^4-27x^2+2x)\,dx}{(x^2-9)}}_{\blacksquare}=\ln C \qquad C\neq 0$$

$\bullet)$ $\displaystyle\int\frac{3y^2}{(y-1)(y^2+y+1)}$

Parc. zlomky

$$3y^2 \overset{!}{=} A(y^2+y+1)+(By+C)(y-1)$$

$$3y^2=(A+B)y^2+y(A-B+C)+(A-C)$$

$A-C=0 \Rightarrow A=C$

$A-B+C=0 \Rightarrow B=2A$

$A+B=3$

$3A=3 \Rightarrow \underline{A=1},\ \underline{C=1}$

$\underline{\underline{B=2}}$

$$=\int\frac{1}{(y-1)}+\int\frac{2y+1}{\underbrace{y^2+y+1}_{t}}$$

$$=\ln(y-1)+\int\frac{1}{t}\,dt$$

$=\ln(y-1)+$ AJO vlastně

$$=\underline{\ln|y^3-1|}$$

$\blacksquare$ $$\int\frac{3x^2(x^2-9)+2x}{(x^2-9)}=\int 3x^2+\int\frac{2x}{(x-3)(x+3)}=x^3+\int\frac{1}{x+3}-\int\frac{1}{x-3}$$

$$=\underline{x^3+\ln|x+3|-\ln|x-3|=x^3+\ln\left|\frac{x+3}{x-3}\right|}$$

$$\ln|y^3-1|=-x^3+\ln\left|\frac{c(x-3)}{x+3}\right|$$

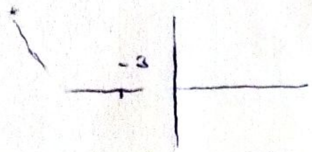
znaménko je v konstantě

$$\underline{y^3-1}=+e^{-x^3}\cdot e^{\ln|\ |}=\underline{e^{-x^3}\frac{c(x-3)}{x+3}}$$

Poč. podmínka $y(1)=\sqrt[3]{2}$

$$2-1=e^{-1}c\frac{(1-3)}{1+3}$$

$$1=\frac{c}{e}\cdot\frac{-2}{4}\Rightarrow \underline{c=-2e}$$



$$\underline{y^3=1+\left(\frac{x-3}{x+3}\right)e^{-(x^3+2)}}$$

problém v bodě -3 nespojitost skoková

$\Rightarrow \underline{I=(-3,+\infty)}$, protože máme poč. podmínku v 1

DIFR 16. 6.

Jan Palán

② $y''' - 2y'' - 5y' + 6y = \sinh 3x = \frac{e^{3x} - e^{-3x}}{2} = \frac{e^{3x}}{2} - \frac{e^{-3x}}{2}$

Najdeme fundamentální systém

$$\lambda^3 - 2\lambda^2 - 5\lambda + 6 \overset{!}{=} 0$$

$$(\lambda-1)(\lambda^2-\lambda-6) = (\lambda-1)(\lambda-3)(\lambda+2) \overset{!}{=} 0$$

$\lambda=1$ $\quad \lambda=3$ $\quad \lambda=-2$

$$FS = \left(e^{x}, e^{-2x}, e^{3x}\right)$$

a) pravá strana

$\frac{e^{3x}}{2}$ $\quad \beta=3 \;\; \gamma=0$ polynomy st. 0

3 je jednonásobným kořenem $\rightarrow k=1$

$y_{p_1} = \underline{x^1 e^{3x}} \cdot \underbrace{a}_{\text{polynom st. 0}}$

$y'_p = ae^{3x} + 3xae^{3x}$

$y''_p = 3ae^{3x} + 3ae^{3x} + 9xae^{3x}$

$y'''_p = 9ae^{3x} + 9ae^{3x} + 9ae^{3x} + 27xae^{3x}$

dosazení do rce

$$\frac{e^{3x}}{2} \overset{!}{=} e^{3x}\left(27a + \cancel{27xa} - 12a - \cancel{18xa} - 5a - \cancel{15xa} + \cancel{6xa}\right)$$

$\frac{1}{2} \overset{!}{=} 10a \qquad \underline{a = \frac{1}{20}} \qquad y_{p_1} = \frac{1}{20} x e^{3x}$

b) pravá strana

$-\frac{e^{-3x}}{2}$ $\quad \beta=-3 \;\; \gamma=0$ st. polynom 0

$y_{p_2} = ae^{-3x} \qquad y'_{p_2} = -3ae^{-3x} \qquad y'' = 9ae^{-3x} \qquad y''' = -27ae^{-3x}$

$$-\frac{1}{2}e^{-3x} \overset{!}{=} e^{-3x}\left(-27a - 18a + 15a + 6a\right)$$

$-\frac{1}{2} \overset{!}{=} -24a$

$a = \frac{1}{48} \qquad y_{p_2} = \frac{1}{48} e^{-3x}$

celkem řešení

$$y(x) = \underbrace{Ae^{x} + Be^{-2x} + Ce^{3x}}_{\text{LK FS}} + \underbrace{\frac{1}{20}xe^{3x} + \frac{1}{48}e^{-3x}}_{\text{partikulární řešení}}$$

③ $y'(x-1)(x-3)+y^2(x-1)-y(3x+1)+2x+6=0$ další řeš. $x=2$

nalézt dílčí řešení

$y=a \qquad a^2(x-1)-3ax-a+2x+6=0$

$a^2-3a+2=0$

$-a^2-a+6=0$

$-4a+8=0$

$\underline{\underline{a=2}}$

(na další řeš. 2 ⇒ musím vydělit další (1,2)

máme 1. řešení $y=2$

a Riccatiho rovnici $\quad y' = -\dfrac{(2x+6)}{(x-1)(x-3)} + \dfrac{y(3x+1)}{(x-1)(x-3)} - y^2 \dfrac{1}{x-3}$

substituce

$y = \underbrace{2}_{\text{řešení}} + \dfrac{1}{z} \qquad y' = -\dfrac{1}{z^2}z' \qquad \dfrac{1}{z} = y-2, \quad z=\dfrac{1}{y-2}$

dosazení

$$-\frac{1}{z^2}z' = -\frac{(2x+6)}{(x-1)(x-3)} + \frac{\left(2+\frac{1}{z}\right)(3x+1)}{(x-1)(x-3)} - \left(2+\frac{1}{z}\right)^2\frac{1}{x-3}$$

$$-\frac{1}{z^2}z' = -\frac{(2x+6)}{(x-1)(x-3)} + \frac{2+6x}{(x-1)(x-3)} + \frac{1}{z}\frac{(3x+1)}{(x-1)(x-3)} - \frac{4(x-1)}{(x-3)(x-1)} - \frac{4}{z}\frac{1}{(x-3)} - \frac{1}{z^2}(x-3)^{-1} \ \Big/ \cdot -z^2$$

$$z' = -z\left(\frac{3x+1}{(x-1)(x-3)}\right) + \frac{4z(x-1)}{(x-1)(x-3)} + \frac{1}{x-3}$$

$$z' = \frac{z\overbrace{(4x-4-3x-1)}^{x-5}}{(x-1)(x-3)} + \frac{1}{x-3}$$

$$z' - z\frac{x-5}{(x-1)(x-3)} = \frac{1}{x-3} \qquad \text{LDR 1. řádu}$$

$$p(x) = \frac{5-x}{(x-1)(x-3)} \qquad P(x) = \int \frac{5-x}{(x-1)(x-3)}$$

$$= \int -\frac{2}{(x-1)} + \int \frac{1}{x-3}$$

$$= -2\ln|x-1| + \ln|x-3|$$

$$= \underline{\underline{\ln\left|\frac{x-3}{(x-1)^2}\right|}}$$

parc. zlomky

$A(x-3)+B(x-1) = 5-x$

$(A+B)x - 3A - B = 5-x$

$A+B=-1$

$-3A-B=5$

$4=-2A$

$A=-2$

$B=1$

Integrační faktor $\quad \underline{\underline{e^{P(x)} = \dfrac{x-3}{(x-1)^2}}}$

Jan Palán

pokračování ③ Jan Palán

IF $\dfrac{x-3}{(x-1)^2}$ $\qquad z' + \dfrac{5-x}{(x-1)(x-3)} z = \dfrac{1}{x-3} \quad / \cdot \text{IF}$

$$z' \frac{x-3}{(x-1)^2} + \frac{(5-x)}{(x-1)^3} z = \frac{1}{(x-1)^2}$$

$$\left(z \frac{x-3}{(x-1)^2} \right)' = \frac{1}{(x-1)^2}$$

$$z \frac{x-3}{(x-1)^2} = \int \frac{1}{(x-1)^2} + C = - \frac{1}{(x-1)} + C$$

$$\frac{1}{y-2} = z = - \frac{(x-1)}{(x-3)} + \frac{C(x-1)^2}{x-3}$$

$$y-2 = \frac{x-3}{C(x-1)^2 - (x-1)} \qquad \underline{\underline{y = 2 + \frac{x-3}{C(x-1)^2 - (x-1)}}}$$

druhé řešení