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B-509

podmínky zápočtu:

a) domácí úkol (+1-10 bodů), $\Sigma > 0$

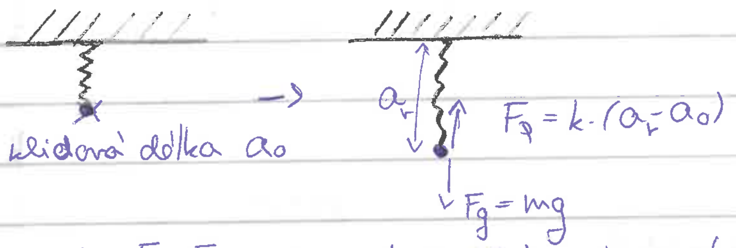
b) presence (2 povolené absence)

Koníček – Tolar – Sbírka řešených příkladů z fyziky III (vlnění)

① hmotný bod na pružině (podléhající Hookovu zákonu)

② matematické kyvadlo

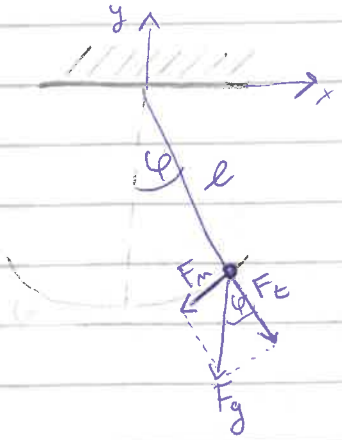
①



$$m\ddot{y} = F_g - F_p = mg - k(a_r - a_0) = mg - k(y(t) - a_0)$$

$$\ddot{y} = g - \frac{k}{m}y(t) + \frac{k}{m}a_0 = -\frac{k}{m}y(t) + g + \frac{k}{m}a_0$$

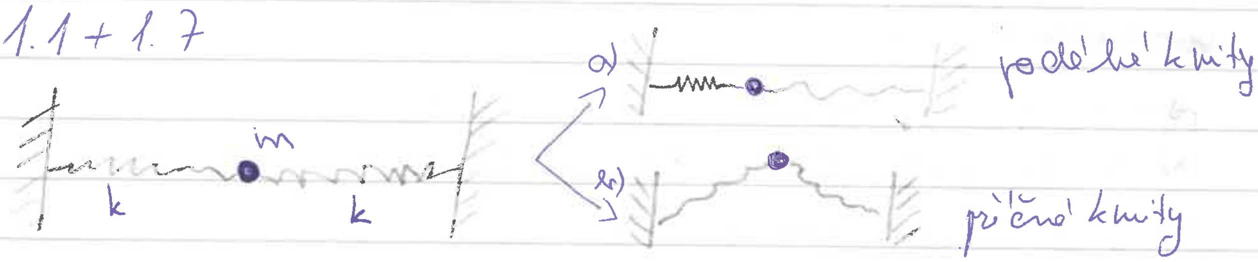
②

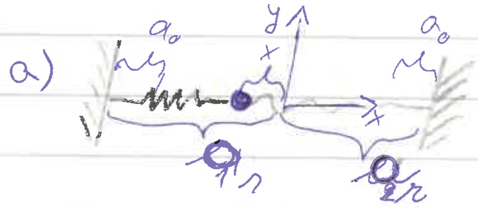


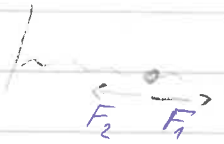
$$x = l\cdot\sin\varphi$$
$$y = -l\cdot\cos\varphi$$
$$F_t = F_g\cos\varphi$$
$$F_n = F_g\sin\varphi$$

$$\left.\begin{aligned} m\ddot{x} &= \dots \\ m\ddot{y} &= \dots \end{aligned}\right\} \left.\begin{aligned} ml\ddot{\varphi} &= -mg\sin\varphi \\ \ddot{\varphi} &= -\frac{g}{l}\sin\varphi \end{aligned}\right\} \xrightarrow{\text{aproximace}}_{\sin x \approx x,\ x\to 0} \ddot{\varphi} + \frac{g}{l}\varphi = 0$$

1.1 + 1.7







$$F_1 + F_2 = 0$$

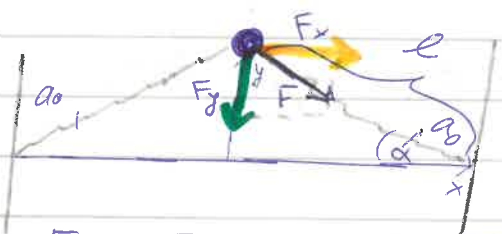
$$k a_1 + k a_1 = 0$$

$$m\ddot{x} = -k(a_1 - a_0 + x) + k(a_1 - a_0 - x)$$

$$m\ddot{x} = -2kx$$

$$\ddot{x} + \underbrace{\frac{2k}{m}}_{\omega^2} x = 0$$

$$x(t) = A \cdot \cos\left(\sqrt{\frac{2k}{m}}\, t + \varphi_0\right)$$



$$F = k(\ell - a_0), \quad \ell = \sqrt{a_1^2 + y^2}$$

$$F_y = -F \cdot \sin\alpha = -k\left(\sqrt{a_1^2+y^2} - a_0\right) \cdot \frac{y}{\sqrt{a_1^2+y^2}}$$

$$F_x = +F \cdot \cos\alpha = -k\left(\sqrt{a_1^2+y^2} - a_0\right) \cdot \frac{a_1}{\sqrt{a_1^2+y^2}}$$

$$m\ddot{x} = 0$$

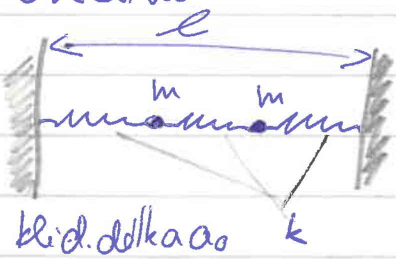
$$m\ddot{y} = -2yk\,\frac{\left(\sqrt{a_1^2+y^2} - a_0\right)}{\sqrt{a_1^2+y^2}} = -2yk\left(1 - \frac{a_0}{\sqrt{a_1^2+y^2}}\right)$$

aproximace:
- dokonalá pružnost: $\ell \gg a_0$, tj. $a_0 \approx 0$ a)
- malé kmity: $|y| \ll a_1$ b)

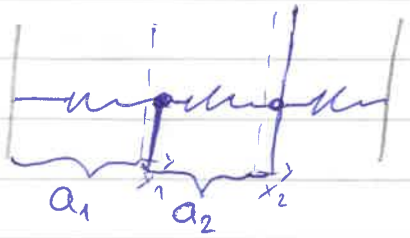
a): $m\ddot{y} = -2ky$

b): $m\ddot{y} = -2k\left(1 - \frac{a_0}{a_1}\right)$

2. cvičení



Výchylky:



$$m\ddot{x}_1 = F_1 \quad , \cancel{a_1 \ldots}$$
$$m\ddot{x}_2 = F_2$$
$$F_1 = -k\cdot(a_1 - a_0) + k(a_2 - a_0) = k(-a_1 + a_2) = (-2x_1 + x_2)$$
$$F_2 = -k(a_2 - a_0) + k(l - a_1 - a_2 - a_0) = k(a_2 + l + a_1 + a_2) = (-2x_2 + x_1)$$

$$a_1 = \tfrac{1}{3} l + x_1$$
$$a_2 = \tfrac{1}{3} l - x_1 + x_2$$

$$m\ddot{x}_1 = k(-2x_1 + x_2)$$
$$m\ddot{x}_2 = k(-2x_2 + x_1)$$

Hledáme řešení ve tvaru módů: $x_1 = A_1 \cos(\omega t + \varphi)$
$x_2 = A_2 \cos(\omega t + \varphi)$

$$-A_1 m\omega^2 \cos(\omega t + \varphi) = k(-2A_1 + A_2)\cos(\omega t + \varphi)$$
$$-A_2 m\omega^2 \cos(\omega t + \varphi) = k(-2A_2 + A_1)\cos(\omega t + \varphi)$$

$$-A_1(m\omega^2 - 2k) - A_2 k = 0$$
$$-A_2(m\omega^2 - 2k) - A_1 k = 0 \quad ; \quad \underbrace{\begin{pmatrix} m\omega^2 - 2k & k \\ k & m\omega^2 - 2k \end{pmatrix}}_{\mathbb{A}}$$

pokud je soustava regulární, pak pouze (0) je řešením

pokud je soustava singulární, pak existuje netriviální řešení $\Leftrightarrow \det \mathbb{A} = 0$

$$0 = \begin{vmatrix} m\omega^2 - 2k & k \\ k & m\omega^2 - 2k \end{vmatrix} =$$
$$= (m\omega^2 - 2k)^2 - k^2 = (m\omega^2 - k)(m\omega^2 - 3k)$$
$$m\omega_1^2 = k$$
$$m\omega_2^2 = 3k$$

$m\omega^2 = k: \begin{pmatrix} -k & k \\ k & -k \end{pmatrix}$, $\ker A = [(1,1)]_\lambda$

$m\omega^2 = 3k: \begin{pmatrix} k & k \\ k & k \end{pmatrix}$, $\ker A = [(1,-1)]_\lambda$

$$\left.\begin{aligned} x_1 &= \alpha \cos\left(\sqrt{\tfrac{k}{m}}\, t + \varphi\right) \\ x_2 &= \alpha \cos\left(\sqrt{\tfrac{k}{m}}\, t + \varphi\right) \end{aligned}\right\}$$

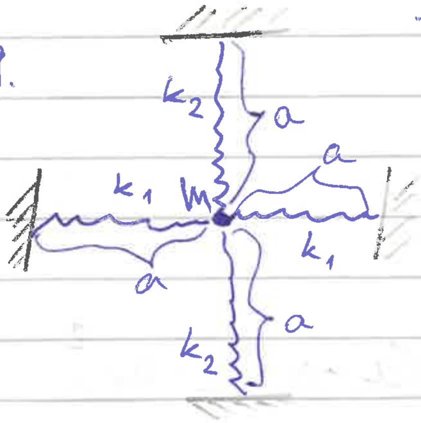
$$\left.\begin{aligned} x_1 &= \beta \cos\left(\sqrt{\tfrac{3k}{m}}\, t + \varphi\right) \\ x_2 &= -\beta \cos\left(\sqrt{\tfrac{3k}{m}}\, t + \varphi\right) \end{aligned}\right\}$$

Hledání módů pomocí vyjádření energie

1. Nalezneme potenciální energii $U(\vec{x})$
2. stabilní rovnovážná poloha $\vec{x}_0$:
   - $\frac{\partial U}{\partial k}(\vec{x}_0) = 0, \forall k$
   - $\frac{\partial^2 U}{\partial x_j \partial x_k}(\vec{x}_0) > 0$
3. nové souřadnice odpovídající výchylkám ze SRP
   - $\vec{\xi} = \vec{x} - \vec{x}_0$
4. $\tilde{U}(\xi) = \frac{1}{2} U_{jk} \xi_j \xi_k$, kde $U_{jk} = \frac{\partial^2 U}{\partial x_j \partial x_k}(\vec{x}_0)$
5. Pohybové rovnice: $m_j \ddot{\xi}_j = -\frac{\partial \tilde{U}}{\partial \xi_j} = -U_{jk} \xi_k$
   - $m_j \ddot{\xi}_j + U_{jk} \xi_k = 0$, $\cancel{T_{jk}} = T = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & \ddots\, m_n \end{pmatrix}$
6. $\det(U - \omega^2 T) = 0 \rightarrow$ frekvence módů
   - řešíme soustavu $(U - \omega_j^2 T)\vec{A} = \vec{0}$, $\forall j \in \hat{n}$
   - automaticky: $A_j^{(m)} T_{jk} A_k^{(\tilde{m})} = 0$, $m \neq \tilde{m}$
   - normalizace $A_j^{(m)} T_{jk} A_k^{(m)} \overset{!}{=} 1 \rightarrow$ normální tvar módů

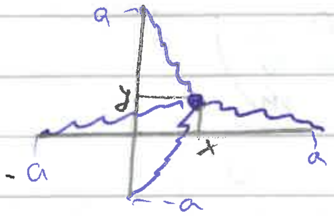
$$\boxed{T_{jk} := \frac{\partial^2 T}{\partial \dot{x}_i \partial \dot{x}_j}(\vec{x}_0)}$$

1.9.



Potenciální energia pružiny

$U = \frac{1}{2} k (l - a_0)^2$



$$U_1 = \frac{1}{2} k_1 \left(\sqrt{(a-x)^2+y^2} - a\right)^2$$
$$U_2 = \frac{1}{2} k_1 \left(\sqrt{(x-a)^2+y^2} - a\right)^2$$
$$U_3 = \frac{1}{2} k_2 \left(\sqrt{(a-y)^2+x^2} - a\right)^2$$
$$U_4 = \frac{1}{2} k_2 \left(\sqrt{(y-a)^2+x^2} - a\right)^2$$

$$U = U_1 + U_2 + U_3 + U_4$$

$$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2)$$

$$\mathbb{T} = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$ Dopočítať doma.

- $\frac{\partial U_1}{\partial x} = \frac{1}{2} k_1 \cdot 2 \left(\sqrt{(a-x)^2+y^2} - a\right)\left(\frac{1}{2}((a-x)^2+y^2)^{-\frac{1}{2}} \cdot (-2(a-x))\right) =$

$= -k_1 (a-x) \cdot \left(1 - \frac{a}{\sqrt{(a-x)^2+y^2}}\right)$

- $\frac{\partial U_1}{\partial y} = \frac{1}{2} k_1 \cdot 2 \left(\sqrt{(a-x)^2+y^2} - a\right) \cdot \left(\frac{1}{2}((a-x)^2+y^2)^{-\frac{1}{2}}\right) \cdot (2y) =$

$= k_1 y \cdot \left(1 - \frac{a}{\sqrt{(a-x)^2+y^2}}\right)$

- $\frac{\partial^2 U_1}{\partial x^2} = -k_1 \Big[(-1) \cdot \left(1 - \frac{a}{\sqrt{(a-x)^2+y^2}}\right) + (a-x) \cdot \Big(-a \cdot \left(-\frac{1}{2}\right)$

$\cdot ((a-x)^2+y^2)^{-\frac{3}{2}} \cdot (2(a-x) \cdot (-1)) =$

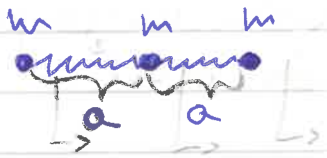
$= \Big|_{x_0 = \binom{0}{0}} = k_1$

- $\frac{\partial^2 U}{\partial y^2} = k_1 \Big[\left(1 - \frac{a}{\sqrt{(a-x)^2+y^2}}\right) + y \cdot \left(+a \frac{1}{2} ((a-x)^2+y^2)^{-\frac{3}{2}}\right) = 0$

- $\frac{\partial^2 U}{\partial x \partial y} = k_1 y \left(+a \frac{1}{2} \cdot ((a-x)^2+y^2)^{-\frac{1}{2}} \cdot (-2(a-x))\right)\Big|_{x_0} = 0$

$\mathbb{U}_1 = \begin{pmatrix} k_1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbb{U}_2 = \begin{pmatrix} k_1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbb{U}_3 = \begin{pmatrix} 0 & 0 \\ 0 & k_2 \end{pmatrix}, \mathbb{U}_4 = \begin{pmatrix} 0 & 0 \\ 0 & k_2 \end{pmatrix}$

1.3



$$U = \tfrac{1}{2}k(\cancel{a} + x_2 - x_1 - \cancel{a})^2 + \tfrac{1}{2}k(\cancel{a} + x_3 - x_2 - \cancel{a})^2$$

$$U = \begin{pmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{pmatrix}, \quad T = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix}$$

$$\det(U - \omega^2 T) = \begin{vmatrix} k - m\omega^2 & -k & 0 \\ -k & 2k - m\omega^2 & -k \\ 0 & -k & k - m\omega^2 \end{vmatrix} = \omega^2 (k - m\omega^2)(3k - m\omega^2) \overset{!}{=} 0$$

frekvence módů: $\omega = 0 \rightarrow \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\omega = \sqrt{\frac{k}{m}} \rightarrow \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$\omega = \sqrt{\frac{2k}{m}} \rightarrow \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

1. mód: $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = q_1 \cos\left(\sqrt{\frac{k}{m}}\, t + \varphi_1\right) \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

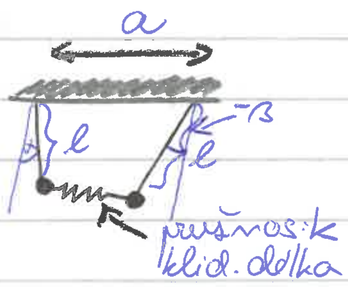
2. mód: $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = q_2 \cos\left(\sqrt{\frac{2k}{m}}\, t + \varphi_2\right) \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

3. mód: translační pohyb

D.Ú. 1.8

Příště: dokončení 1. kapitoly
nemechanické kmitání

1.5.



g ↓

① módy

② časový vývoj s počáteční podm.

$t=0$: $x_1 = x_2 = A$

$\dot{x}_1 + \dot{x}_2 = 0$

$x_1 = l\sin\alpha \qquad x_2 = l\sin\beta + a$

$y_1 = -l\cos\alpha \qquad y_2 = -l\cos\beta$

$$U = mgy_1 + mgy_2 + \frac{1}{2}k(\overbrace{x_2 - x_1 - a}^{\text{platné pro malé } \alpha,\beta \ (\text{zanedbatelné } |y_2-y_1| \ll |x_2-x_1|)})^2 =$$
$$= -mgl(\cos\alpha + \cos\beta) + \frac{1}{2}kl^2(\sin\beta - \sin\alpha)^2$$

m

$$T = \frac{1}{2}m(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m(\dot{x}_2^2 + \dot{y}_2^2) = \frac{1}{2}ml^2(\dot{\alpha}^2 + \dot{\beta}^2)$$

$$\frac{\partial U}{\partial \alpha} = +mgl\sin\alpha - \frac{1}{2}kl^2 \cdot 2(\sin\beta - \sin\alpha)\cdot\cos\alpha$$
$$\frac{\partial U}{\partial \beta} = mgl\sin\beta - \frac{1}{2}kl^2\cdot 2(\sin\alpha - \sin\beta)\cos\beta$$

R.P. pro $\alpha = \beta = 0$

$$\frac{\partial^2 U}{\partial \alpha^2} = mgl\cos\alpha - kl^2(-\sin\alpha(\sin\beta - \sin\alpha) + \cos\alpha(-\cos\alpha)) =$$
$$= mgl\cos\alpha + kl^2\sin\alpha(\sin\beta - \sin\alpha) + kl^2\cos^2\alpha$$

$$\frac{\partial^2 U}{\partial \beta^2} = mgl\cos\beta + kl^2\sin\beta(\sin\alpha - \sin\beta) + kl^2\cos^2\beta$$

$$\frac{\partial^2 U}{\partial \alpha\partial\beta} = +kl^2\cos\alpha\cos\beta$$

$$U = \begin{pmatrix} mgl + kl^2 & -kl^2 \\ -kl^2 & mgl + kl^2 \end{pmatrix}$$

$$T = \begin{pmatrix} ml^2 & 0 \\ 0 & ml^2 \end{pmatrix} \qquad \frac{\partial^2 T}{\partial \dot{\alpha}^2} = ml^2$$
$$\frac{\partial^2 T}{\partial \dot{\beta}^2} = ml^2$$

$$\det(\mathbb{V} - \omega^2 \mathbb{T}) = (mgl + 2kl^2 - ml^2\omega^2)(mgl - ml^2\omega^2) \overset{!}{=} 0$$

$$\omega_1^2 = \frac{g}{l} + \frac{2k}{m} \quad \dots \quad \left[\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right]_\lambda$$

$$\omega_2^2 = \frac{g}{l} \quad \dots \quad \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]_\lambda$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = A_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cos(\omega_1 t + \varphi_1) + A_2 \cos(\omega_2 t + \varphi_2) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = l A_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cos(\omega_1 t + \varphi_1) + l A_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cos(\omega_2 t + \varphi_2)$$

Uvažujme slabou pružinu: $K \ll \frac{mg}{l}$

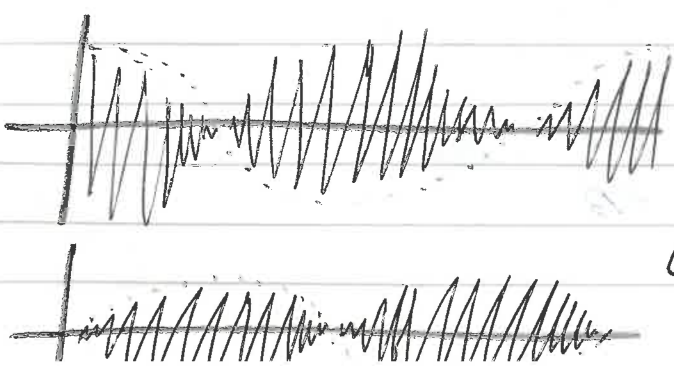
$$\omega_1 \gtrsim \omega_2$$

$$x_1 = A\cos(\omega_1 t) + A\cos(\omega_2 t)$$
$$x_2 = A\cos(\omega_1 t) - A\cos(\omega_2 t)$$

$$x_1 = 2A\cos\left(\frac{\omega_1 + \omega_2}{2} t\right)\cos\left(\frac{\omega_1 - \omega_2}{2} t\right)$$
$$x_2 = 2A\sin\left(\frac{\omega_1 + \omega_2}{2} t\right)\sin\left(\frac{\omega_2 - \omega_1}{2} t\right)$$

$$\omega_+ := \frac{\omega_1 + \omega_2}{2}$$
$$\omega_- := \frac{\omega_2 - \omega_1}{2}$$

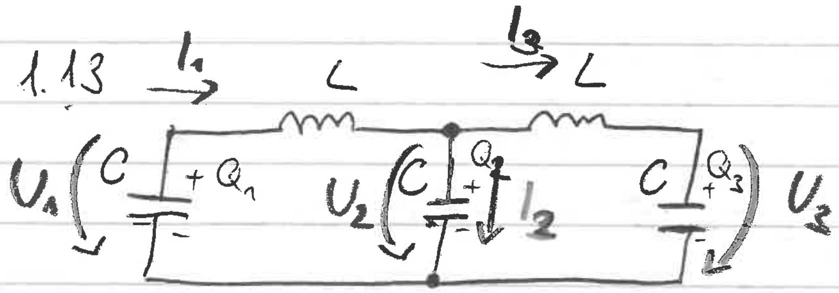
$$x_1 = 2A\cos(\omega_- t)\cos(\omega_+ t)$$
$$x_2 = 2A\sin(\omega_- t)\sin(\omega_+ t)$$



Úhlový kmitočet rázů $\cdot\, 2\omega_-$

$$E_1 = \frac{1}{2} m\omega_+^2 (A\cos(\omega_- t))^2$$
$$E_2 = \frac{1}{2} m\omega_+^2 (A\sin(\omega_- t))^2$$

1.13



$I_1 = -\dot{Q}_1$

$I_2 = +\dot{Q}_2$

$I_3 = +\dot{Q}_3$

Řešení pomocí energií

$E_C = \frac{1}{2} C U^2 = \frac{1}{2C} Q^2$ Kirchhoffův zákon:

$E_L = \frac{1}{2} L I^2$ $I_1 = I_2 + I_3$

$$E = E_{L1} + E_{L2} + E_{C1} + E_{C2} + E_{C3}$$

$$E = \underbrace{\frac{1}{2} L \dot{Q}_1^2 + \frac{1}{2} L \dot{Q}_3^2}_{\text{„}T\text{"}} + \underbrace{\frac{1}{2C} Q_1^2 + \frac{1}{2C} Q_2^2 + \frac{1}{2C} Q_3^2}_{\text{„}U\text{"}}$$

$\boxed{\text{Zjednodušení:}}$

$$-\dot{Q}_1 = \dot{Q}_2 + \dot{Q}_3$$

$$Q_2 = -Q_1 - Q_3 + C,$$

zvolme $C = 0$: $Q_2 = -Q_1 - Q_3$

$$T = \frac{1}{2} L \dot{Q}_1^2 + \frac{1}{2} L \dot{Q}_3^2 = \frac{1}{2} L (\dot{Q}_1^2 + \dot{Q}_3^2)$$

$$U = \frac{1}{2C} (Q_1^2 + (Q_1 + Q_3)^2 + Q_3^2) =$$

$$= \frac{1}{2C} (2Q_1^2 + 2Q_1 Q_3 + 2Q_3^2) = \frac{1}{C} (Q_1^2 + Q_1 Q_3 + Q_3^2)$$

$$\frac{\partial U}{\partial Q_1} = \frac{1}{C} (2Q_1 + Q_3), \quad \frac{\partial^2 U}{\partial Q_1^2} = \frac{2}{C}$$

$$\frac{\partial U}{\partial Q_3} = \frac{1}{C} (2Q_3 + Q_1), \quad \frac{\partial^2 U}{\partial Q_3^2} = \frac{2}{C}$$

$$\frac{\partial^2 U}{\partial Q_1 \partial Q_3} = \frac{1}{C} \qquad \mathbb{U} = \frac{1}{C} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\frac{\partial T}{\partial \dot{Q}_1} = \frac{1}{2} L \cdot 2 \dot{Q}_1, \quad \frac{\partial^2 T}{\partial \dot{Q}_1^2} = \cancel{2} L, \quad \mathbb{T} = \begin{pmatrix} L & 0 \\ 0 & L \end{pmatrix}$$

$$\det(U-\omega^2 T) = \begin{vmatrix} \frac{2}{C}-L\omega^2 & \frac{1}{C} \\ \frac{1}{C} & \frac{2}{C}-L\omega^2 \end{vmatrix} =$$

$$= \left(\frac{2}{C}-L\omega^2\right)^2 - \frac{1}{C^2} = \left(\frac{2}{C}-L\omega^2-\frac{1}{C}\right)\left(\frac{2}{C}-L\omega^2+\frac{1}{C}\right) =$$

$$= \left(\frac{1}{C}-L\omega^2\right)\left(\frac{3}{C}-L\omega^2\right)$$

$$\omega_1^2 = \frac{3}{LC}, \quad \omega_2^2 = \frac{1}{LC}$$

Vlnění

- spojitá limita kmitání

$$A\frac{\partial^2 \xi}{\partial t^2} = B\frac{\partial^2 \xi}{\partial x^2}$$

předpokládáme řešení tvaru:

$$\xi(x,t) = e^{at+bx} = e^{at}\cdot e^{bx}$$

BÚNO: $a \to i\omega$ ← úhlová frekvence
$b \to ik$ ← vlnové číslo

Dosazení: $\rho\omega^2 = T\cdot k^2$

$$|\omega| = \sqrt{\frac{T}{\rho}}|k|$$

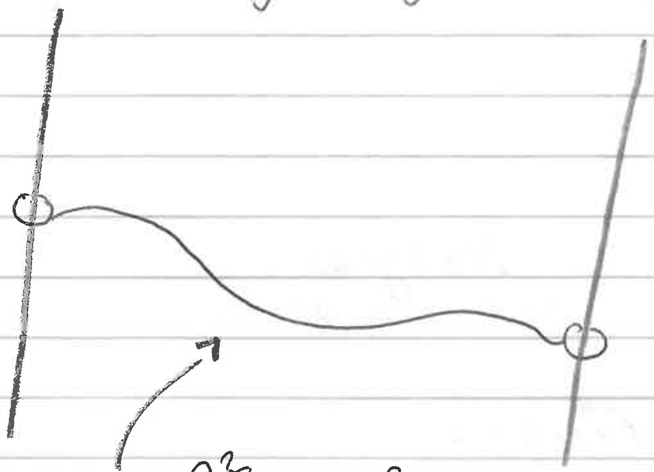
Obecně $\omega = \omega(k)$ – disperzní vztah

- Reálná řešení:

$$\xi(x,t) = \cos(\omega t + kx), \quad \left(k = \sqrt{\frac{\rho}{T}}\,\omega\right)$$

2.9.

Příčné kmity struny s volnými konci



$$\frac{\partial \xi}{\partial x}(0,t)=0$$

$$\frac{\partial \xi}{\partial x}(L,t)=0,\ \forall t$$

$$\rho \frac{\partial^2 \xi}{\partial t^2} = T\frac{\partial^2 \xi}{\partial x^2}$$

Hledejme řešení ve tvaru $\xi(x,t) = f(x)g(t)$

7. cvičení

Módy napjaté struny, Fourierova analýza

$$\rho_L \frac{\partial^2 \psi}{\partial t^2} = T \frac{\partial^2 \psi}{\partial z^2}, \quad \rho_L, T > 0$$

Řešení tvaru: $\psi(z,t) = f(z)\, g(t)$

$$\rho_L\, g''(t)\, f(z) = T\, f''(z)\, g(t)$$

$$\rho_L \frac{g''(t)}{g(t)} = T \frac{f''(z)}{f(z)} = \underset{\text{const.}}{C}$$

$$\Rightarrow \quad T f''(z) = C f(z)$$
$$\rho_L\, g''(t) = C\, g(t)$$

Harmonický pohyb $\Leftrightarrow C < 0$

$$\left.\begin{aligned} f(z) &= A\cos\Big(\overbrace{\sqrt{\tfrac{C}{T}}}^{k}\, z + \varphi_0\Big) \\ g(t) &= B\cos\Big(\underbrace{\sqrt{\tfrac{C}{\rho}}}_{\omega}\, t + \chi_0\Big) \end{aligned}\right\} \text{stojaté vlnění}$$

Frekvence: $\omega = \sqrt{\frac{C}{\rho}}$

vlnové číslo: $k = \sqrt{\frac{C}{T}} = \sqrt{\frac{\rho_L}{T}}\,\omega$, $\quad v = \frac{\omega}{k} = \sqrt{\frac{T}{\rho_L}}$

Závislost mezi $\underline{\omega}$ a $\underline{k}$ = disperzní vztah

Postupné vlnění:

$$\psi(z,t) = A\cos(\omega t - kz + \varphi_0) =$$
$$= A\cos\underbrace{(\cancel{\omega t})}_{\omega t + \varphi_0}\cos(kz) + A\sin(\omega t + \varphi_0)\sin(kz)$$

(stojaté + stojaté)

Opačný přechod:

stojatá: $\psi(z,t) = A\cos(kz+\varphi_0)\cos(\omega t+\chi_0)$

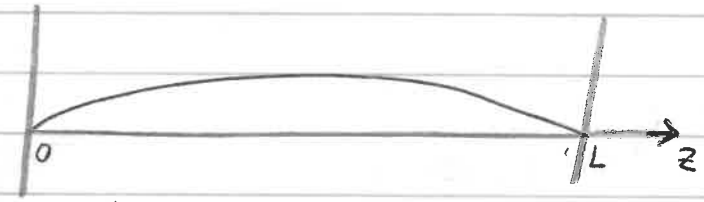
$= \frac{A}{2}\left(\cos(kz+\omega t+\varphi_0+\chi_0) - \cos(kz-\omega t-\varphi_0-\chi_0)\right)$

$\longleftarrow \qquad\qquad \longrightarrow$

(postupná + postupná!)

Význam okrajových podmínek

$\psi(z,t) = A\cos(kz+\varphi_0)\cos(\omega t+\chi_0)$ + omezení



$\left.\begin{array}{l}\psi(0,t)=0\\ \psi(L,t)=0\end{array}\right\}$ pevné konce

$\left.\frac{\partial}{\partial z}\psi(z,t)\right|_{z=0} = 0$

$\left.\frac{\partial}{\partial z}\psi(z,t)\right|_{z=L} = 0$

Pro oba pevné konce: $A\sin\left(\frac{\pi n}{L}z\right)\cos\left(\sqrt{\frac{T}{\rho}}\frac{\pi n}{L}t+\varphi_0\right)$

$n\in\mathbb{N}$

$k = \frac{\pi n}{L} \Rightarrow \omega = \sqrt{\frac{T}{\rho}}\cdot\frac{\pi n}{L}$

Řešení počátečních podmínek

Obecné kmitání struny s pevnými konci:

$\psi(z,t) = \sum_{n=1}^{\infty} A_n \sin\Big(\underbrace{\frac{\pi n}{L}z}_{kz=\frac{\pi n}{L}z}\Big)\cos\Big(\underbrace{v\frac{\pi n}{L}t}_{\omega t = kvt = k\sqrt{\frac{T}{\rho_L}}t}+\varphi_n\Big)$

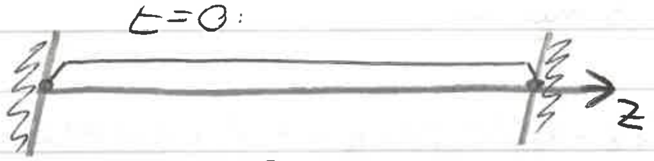
+ omezení

Ekvivalentně:

$$\Psi(z,t)=\sum_{n=1}^{\infty}\left(\tilde{A}_n \sin\left(\frac{\pi n}{L}z\right)\cos\left(\frac{\pi n v}{L}t\right)+\tilde{B}_n \sin\left(\frac{\pi n}{L}z\right)\sin\left(\frac{\pi n v}{L}t\right)\right)$$

Omezení $\to$ hodnoty A_n, φ_n nebo $\tilde{A}_n, \tilde{B}_n$
$\Rightarrow$ jednoznačné řešení $\Psi(z,t)$

2,7: $t=0$:



Lk módů?

$$\Psi(z,0)=f(z)=\begin{cases}0, & z=0 \vee z=L\\ a, & z\in(0,L)\end{cases}$$

$$\sum_{n=1}^{\infty}\left(\tilde{A}_n \cos\left(\frac{\pi n}{L}z\right)\sin\left(\frac{\pi n v}{L}t\right)+\tilde{B}_n \sin\left(\frac{\pi n}{L}z\right)\sin\left(\frac{\pi n v}{L}t\right)\right)$$

$$t=0 \quad \sum_{n=1}^{\infty}\tilde{A}_n \cos\left(\frac{\pi n}{L}z\right)=f(z)$$

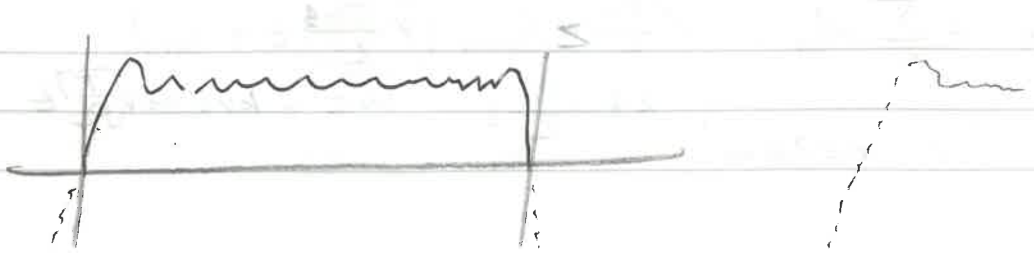
Řešení: ~~Skts~~

$$\tilde{A}_n=\frac{2}{L}\int_0^L f(z)\sin\left(\frac{\pi n z}{L}\right)dz, \quad f(z)=a, \ \forall z\in(0,L)$$

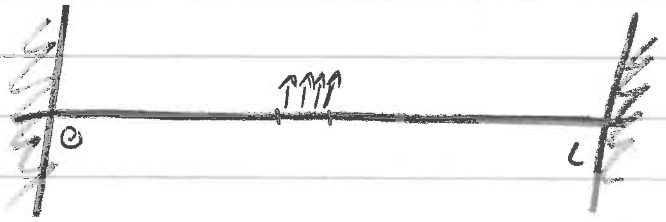
$$\Rightarrow \tilde{A}_n=\frac{2}{L}\int_0^L a\cdot\sin\left(\frac{\pi n}{L}z\right)dz=\dots=\circledast$$

$$\circledast=\frac{2a}{\pi n}\left((-1)^{n+1}+1\right)=\begin{cases}0, & n \text{ sudé}\\ \frac{4a}{\pi n}, & n \text{ liché}\end{cases}$$

$$\boxed{f(z)=\sum_{n=1}^{\infty}\frac{4a}{\pi(2n+1)}\sin\left(\frac{\pi z}{L}(2n+1)\right)}$$



2.13



~~$t=0: \psi(z,0) = f(z)$~~

$t = 0: \quad \psi(z,0) = 0$

$$\frac{\partial \psi}{\partial t}(z,0) = \begin{cases} v_0, & z \in \left(\frac{L}{2} - \frac{d}{2}, \frac{L}{2} + \frac{d}{2}\right) \\ 0, & \text{jinak} \end{cases}$$

$$\psi(z,t) = \sum_{n=1}^{\infty} \tilde{A}_n \sin\left(\frac{\pi n}{L} z\right) \cos\left(\frac{\pi n v}{L} t\right) + \tilde{B}_n \sin\left(\frac{\pi n}{L} z\right) \sin\left(\frac{\pi n v}{L} t\right)$$

- $\psi(z,0) = \sum_{n=1}^{\infty} \tilde{A}_n \sin\left(\frac{\pi n}{L} z\right) = 0 \Rightarrow \tilde{A}_n = 0, \ \forall n \in \mathbb{N}$

- $\frac{\partial \psi}{\partial t} = \sum_{n=1}^{\infty} \tilde{B}_n \sin\left(\frac{\pi n z}{L}\right) \underbrace{\cos\left(\frac{\pi n v}{L} t\right)}_{1} \frac{\pi n v}{L} \stackrel{!}{=} \underbrace{\begin{cases} v_0, & z \in \left(\frac{L}{2} - \frac{d}{2}, \frac{L}{2} + \frac{d}{2}\right) \\ 0, & \text{jinak} \end{cases}}_{f(z)}$

$$\tilde{B}_n \frac{\pi n v}{L} = \frac{2}{L} \int_0^L f(z) \sin\left(\frac{\pi n}{L} z\right) dz$$

$$= \frac{2}{L}\left[\underbrace{\int_0^{\frac{L}{2}-\frac{d}{2}} 0 \sin\left(\frac{\pi n z}{L}\right) dz}_{0} + \int_{\underbrace{\frac{L}{2}-\frac{d}{2}}_{a}}^{\overbrace{\frac{L}{2}+\frac{d}{2}}^{b}} v_0 \sin\left(\frac{\pi n z}{L}\right) dz + \underbrace{\int_{\frac{L}{2}+\frac{d}{2}}^{L} 0 \sin\left(\frac{\pi n z}{L}\right) dz}_{0}\right] =$$

$$= \frac{2\pi n v}{L^2} \int_a^b v_0 \sin\left(\frac{\pi n z}{L}\right) dz = \cancel{\frac{2\pi n v}{L^2} \frac{L}{\pi n}}$$

$$= 2\pi \frac{n v v_0}{L^2} \underbrace{\left[-\cos\left(\frac{\pi n}{2L}(L+d)\right) + \cos\left(\frac{\pi n}{2L}(L-d)\right)\right]}_{-\cos\alpha \ + \cos\beta} =$$

$$= -2\pi \frac{n v v_0}{L^2} \left(2 \sin\left(\frac{\pi n}{2}\right) \sin\left(\frac{\pi n d}{2L}\right)\right) =$$

$=$

špatně! opravit doma!

5. cvičení (31. 10)

2.28

zákl. tón ≃ první mód

$L = 1\,m$

d (průměr) $= 0{,}5\,mm$

$\nu = 256\,Hz$

$\rho = 9\,g\,cm^{-3}$ – $\frac{\text{hmotnost}}{\text{objem}}$

$T = ?$

$v = \sqrt{\frac{T}{\rho_\ell}}$ ← $\frac{\text{hmotnost}}{\text{délka}}$

$\omega = vk$

$k = \frac{2\pi}{\lambda}, \quad \lambda = 2L$

$$\rho_\ell = \rho_V \cdot S = \rho_V \frac{\pi d^2}{4}$$

$$T = v^2 \cdot \rho_\ell = \left(\frac{\omega}{k}\right)^2 \rho_V \frac{\pi d^2}{4} =$$

$$= \left(\frac{\omega}{\frac{\pi}{L}}\right)^2 \rho_V \frac{\pi d^2}{4} = \left(\frac{2\pi \nu L}{\pi}\right)^2 \rho_V \frac{\pi d^2}{4} =$$

$$= 4\nu^2 L^2 \rho_V \pi \frac{d^2}{4} = \rho_V \nu^2 L^2 d^2 \pi$$

2.5

$$\psi_1 = A_1 \cos(\omega t - kz + \varphi_1)$$
$$\psi_2 = A_2 \cos(\omega t - kz + \varphi_2)$$

$$\psi_1 + \psi_2 = (A_1 \cos(\omega t - kz + \varphi_1) + A_2 \cos(\omega t - kz + \varphi_2)) =$$

$$= Re\left(A_1 e^{i(\omega t - kz + \varphi_1)} + A_2 e^{i(\omega t - kz + \varphi_2)}\right) =$$

$$= Re\left(e^{i(\omega t - kz)} \underbrace{(A_1 e^{i\varphi_1} + A_2 e^{i\varphi_2})}_{A e^{i\varphi}}\right) =$$

$$A_1 e^{i\varphi_1} + A_2 e^{i\varphi_2} = A_1\cos\varphi_1 + A_2\cos\varphi_2 + i(A_1\sin\varphi_1 + A_2\sin\varphi_2)$$

$$\Rightarrow \cancel{Re(A_1 e^{i\varphi_1} + A_2 e^{i\varphi_2}) = A_1\cos\varphi_1 + A_2\cos\varphi_2}$$

$$Re\big((\cos(\omega t - kz) + i\sin(\omega t - kz))(A_1\cos\varphi_1 + A_2\cos\varphi_2 + i(A_1\sin\varphi_1 + A_2\sin\varphi_2))\big)$$

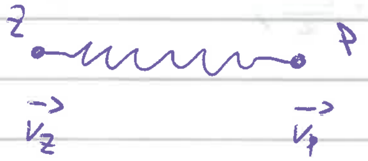
$$= \underbrace{(A_1\cos\varphi_1 + A_2\cos\varphi_2)}_{\tilde A}\cos(\omega t - kz) - \underbrace{(A_1\sin\varphi_1 + A\sin\varphi_2)}_{\tilde B}\sin(\omega t - kz) =$$

$$= \cancel{\tilde A \cup \tilde B}\ \tilde A\cos(\omega t - kz) - \tilde B\sin(\omega t - kz) =$$

$$= A\cos(\omega t - kz + \varphi)$$

$$A = \sqrt{\tilde A^2 + \tilde B^2}, \quad \varphi = \operatorname{arctg}\frac{\tilde B}{A}$$

2.24 Dopplerův jev



V prostředí (ve stojatých souřadnicích) $\omega = k \cdot c$

$$\psi(x,t) = A\cos(\omega t - kx + \varphi_0) = A\cos(\omega t - \tfrac{\omega}{c}x + \varphi_0)$$

a) zdroj se pohybuje $x_z = v_z t$

V místě zdroje: $\psi(x_z, t) = A\cos(\omega t - \frac{\omega}{c}v_z t + \varphi_0)$

$$= A\cos\big(\underbrace{\omega(1 - \tfrac{v_z}{c})}_{\omega_0} t + \varphi_0\big)$$

b) pozorovatel se pohybuje (zdroj v klidu)

v místě zdroje: $\psi(0,t) = A\cos(\omega t + \varphi) \overset{!}{=} A_0\cos(\omega_0 t + \varphi_0)$

v místě poz: $\psi(v_p t, t) = A\cos(\omega_0 t - \frac{\omega_0}{c}v_p t + \varphi_0) =$

$$= A\cos(\omega_0(1 - \tfrac{v_p}{c})t + \varphi_0) = A\cos(\omega_p t + \varphi_0)$$

Fourierova analýza

- funkce na konečném intervalu $(0, L)$

$x \in (0, L)$:

$$\sum_{n=0}^{\infty} a_n \cos \frac{\pi n x}{L} \text{ nebo } \sum_{n=1}^{\infty} b_n \sin \frac{\pi n x}{L}$$

- periodická $f(x)$ s periodou $\lambda \to \sum_{n=0}^{\infty} a_n \cos \frac{2\pi n x}{\lambda} + \sum_{n=1}^{\infty} b_n \sin \frac{2\pi n x}{\lambda}$

- obecná neperiodická funkce $f(t): \mathbb{R} \to \mathbb{R}$

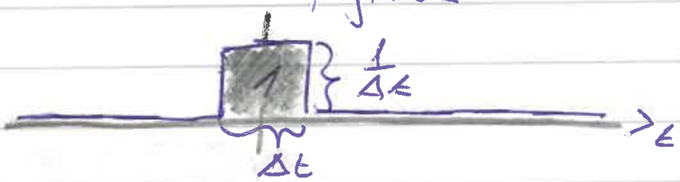
$$\int_0^{+\infty} (a(\omega)\cos(\omega t) + b(\omega)\sin(\omega t))\,d\omega$$

$$a(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} f(t)\cos(\omega t)\,dt$$

$$b(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} f(t)\sin(\omega t)\,dt$$

4 ill

$$f(t) = \begin{cases} \frac{1}{\Delta t} & \text{pro } t \in \left(-\frac{\Delta t}{2}, +\frac{\Delta t}{2}\right) \\ 0 & \text{jinak} \end{cases}$$



$$a(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} f(t)\cos(\omega t)\,dt = \frac{1}{\pi}\int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} \frac{1}{\Delta t}\cos(\omega t)\,dt =$$

$$= \frac{1}{\pi \Delta t \omega}\left[\sin(\omega t)\right]_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} =$$

$$= \frac{2}{\pi \omega \Delta t} \sin \frac{\Delta t\, \omega}{2}$$

$$b(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} f(t)\cos(\omega t)\,dt = \frac{1}{\pi}\int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} \frac{1}{\Delta t}\cos(\omega t)\,dt = 0$$

4.13

$$f(t) = \begin{cases} e^{-\frac{t}{2\tau}} \cos \omega_0 t, & t > 0 \\ 0, & t < 0 \end{cases}$$

$$a(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} f(t)\cos(\omega t)\,dt = \frac{1}{\pi}\int_{0}^{+\infty} e^{-\frac{t}{2\tau}}\cos(\omega_0 t)\cos(\omega t)\,dt =$$

$$= \frac{1}{2\pi}\int_{0}^{+\infty} e^{-\frac{t}{2\tau}}\left(\cos(\omega+\omega_0)t + \cos(\omega_0-\omega)t\right)dt =$$

$=$

4.29

4.30

4.31

4.32

4.33

6. cvičení

Disperze:
Šíření vln různými rychlostmi pro různé k (nebo λ nebo ω)

Disperzní vztah: $\omega = \omega(k)$

- fázová rychlost: $v_\varphi(k) = \frac{\omega(k)}{k}$
- grupová rychlost: $v_g(k) = \frac{d\omega}{dk}$

4.1/
$\omega = v \cdot k \quad , v = konst$

$$V_\varphi = \frac{v \cdot k}{k} = v$$
$$V_g = \frac{d}{dk}(v \cdot k) = v$$

4.3/ V prostředí (sklo, vzduch, voda)

$$k(\omega) = \frac{\omega}{c}\left(1 + \frac{Ne^2}{2m\varepsilon_0} \cdot \frac{1}{\omega_0^2 - \omega^2}\right)$$

- $v_\varphi = \frac{c}{\left(1 + \frac{Ne^2}{2m\varepsilon_0} \frac{1}{\omega_0^2 - \omega^2}\right)}$

- v_g: $\frac{1}{v_g} = \frac{dk}{d\omega} = \frac{1}{c}(\ldots) + \frac{\omega}{c}\left(\frac{Ne^2}{2m\varepsilon_0} \frac{-2\omega}{(\omega_0^2 - \omega^2)^2}\right) =$

$$= \frac{1}{c}\left[\left(1 + \frac{Ne^2}{2m\varepsilon_0} \frac{1}{\omega_0^2 - \omega^2}\right) + \left(\frac{Ne^2}{2m\varepsilon_0} \frac{-2\omega^2}{(\omega_0^2 - \omega^2)^2}\right)\right] =$$

$$= \frac{1}{c} + \frac{Ne^2}{2m\varepsilon_0 c} \frac{\omega_0^2 - \omega^2 - 2\omega^2}{(\omega_0^2 - \omega^2)^2}$$

4.4: V plazmatu: $\omega^2 = k^2c^2 + \omega_p^2$

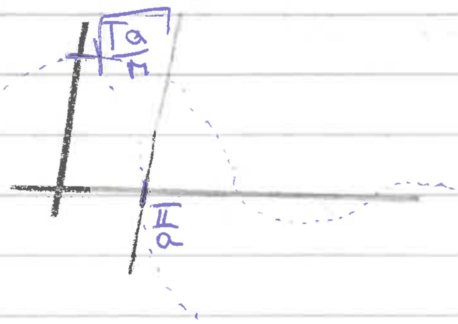
$$\psi_1(z,t) = A\cos(\omega_1 t - k_1 z)$$
$$\psi_2(z,t) = A\cos(\omega_2 t - k_2 z)$$

$$k_1 = k_0 + \Delta k \implies \omega_1 = \omega(k_0+\Delta k) = \omega_0 + \frac{d\omega}{dk}\Delta k + O(\Delta k^2)$$
$$k_2 = k_0 - \Delta k \qquad \omega_2 = \omega(k_0-\Delta k) = \omega_0 - \frac{d\omega}{dk}\Delta k + O(\Delta k^2)$$

$$\psi(z,t) = \psi_1 + \psi_2 = A\cos\left(\omega_0 t - k_0 z + \frac{d\omega}{dk}\Delta k t - \Delta k z\right) +$$
$$+ A\cos\left(\omega_0 t - k_0 z - \frac{d\omega}{dk}\Delta k t + \Delta k z\right) =$$
$$= 2A\cos(\omega_0 t - k_0 z)\cos\left(\frac{d\omega}{dk}\Delta k t - \Delta k z\right) =$$
$$= 2A\underbrace{\cos\left(-k_0\left(z - \frac{\omega_0}{k_0}t\right)\right)}_{\text{nosná vlna } v_\varphi}\underbrace{\cos\left(-\Delta k\left(z - \frac{d\omega}{dk}t\right)\right)}_{\text{obálka } v_g}$$

4.6 | Struna se soustředěnými hmotnostmi

$$\omega = 2\sqrt{\frac{T}{Ma}}\sin\frac{ka}{2}$$



$$v_\varphi = 2\sqrt{\frac{T}{Ma}}\,\frac{\sin\frac{ka}{2}}{k}$$

$$\lim_{k\to 0} v_\varphi(k) = \sqrt{\frac{Ta}{M}}$$

$$v_g = 2\sqrt{\frac{T}{Ma}}\cos\frac{ka}{2}\cdot\frac{a}{2}$$

$$\lim_{k\to 0} v_g(k) = \sqrt{\frac{T}{Ma}}$$

Platí: $\omega = kc \Longleftrightarrow v_\varphi = v_g = c$, $c = \text{konst}$

4.15: Ukažte, že v prostředí s $n = n(\lambda)$ platí:

$$\frac{1}{v_g} = \frac{1}{v_\varphi} - \frac{1}{c}\lambda\frac{dn}{d\lambda}$$

index lomu: $n = \frac{c}{v_\varphi}$

vlnová délka: automaticky ve vakuu, tj: $\lambda = \frac{2\pi}{k_0}$,
kde $\omega = k_0 c$, tj: $k_0 = \frac{\omega}{c}$, $\lambda = \frac{2\pi c}{\omega}$

$$\frac{1}{v_g} = \frac{dk}{d\omega} \underset{k = \frac{\omega}{v_\varphi}}{=} \frac{d}{d\omega}\left(\frac{\omega}{v_\varphi}\right) \underset{v_\varphi = \frac{c}{n}}{=} \frac{d}{d\omega}\left(\frac{\omega n}{c}\right) =$$

$$= \frac{1}{c}\frac{d}{d\omega}(\omega n) = \frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega} =$$

$$= \frac{1}{v_\varphi} + \frac{\omega}{c}\frac{dn}{d\lambda}\frac{d\lambda}{d\omega} =$$

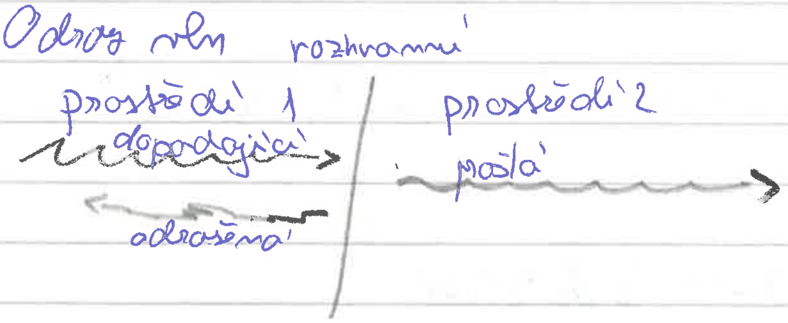
$$= \frac{1}{v_\varphi} + \frac{dn}{d\lambda}\cdot\frac{\omega}{c}\cdot\frac{-2\pi c}{\omega^2} = \frac{1}{v_\varphi} - \frac{dn}{d\lambda}\underbrace{\frac{2\pi c}{\omega}}_{\lambda}\frac{1}{c} =$$

$$= \frac{1}{v_\varphi} - \frac{dn}{d\lambda}\lambda\frac{1}{c}$$

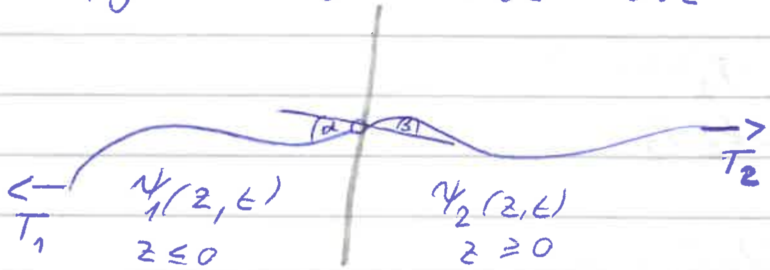
4.9
4.10
4.27
4.28

7. cvičení

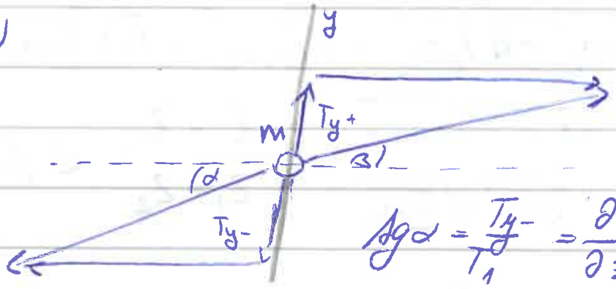
Odraz vln



Napjaté struny: $\rho_j \dfrac{\partial^2 \psi}{\partial t^2} = T_j \dfrac{\partial^2 \psi}{\partial z^2} \Rightarrow \omega_j^2 = \dfrac{T}{\rho} k_j^2$



$$\psi_1(0,t) = \psi_2(0,t)$$



$$\operatorname{tg}\alpha = \frac{T_{y-}}{T_1} = \left.\frac{\partial \psi_1}{\partial z}\right|_{z=0}$$

$$\operatorname{tg}\beta = \frac{T_{y+}}{T_2} = \left.\frac{\partial \psi_2}{\partial z}\right|_{z=0}$$

zrychlení korálku $= \left.\dfrac{\partial^2 \psi}{\partial t^2}\right|_{z=0}$

$$m \left.\frac{\partial^2 \psi}{\partial t^2}\right|_{z=0} = T_{y+} - T_{y-} = T_2 \left.\frac{\partial \psi_2}{\partial z}\right|_{z=0} - T_1 \left.\frac{\partial \psi_1}{\partial z}\right|_{z=0}$$

$$\lim_{m \to 0} \quad 0 = T_2 \left.\frac{\partial \psi_2}{\partial z}\right|_{z=0} - T_1 \left.\frac{\partial \psi_1}{\partial z}\right|_{z=0}$$

$$\boxed{\begin{aligned} &\psi_1(0,t) = \psi_2(0,t), && \forall t \in \mathbb{R} \\ &T_2 \frac{\partial \psi_1}{\partial z}(0,t) = T_1 \frac{\partial \psi_2}{\partial z}(0,t), && \forall t \in \mathbb{R} \end{aligned}}$$

$$\psi_1(z,t) = \underbrace{A \cos(\omega t - k_1 z)}_{\text{dopadající}} + \underbrace{R \cdot A \cos(\omega t + k_1 z)}_{\text{odražená}} \qquad k_1 = \sqrt{\frac{\rho_1}{T_1}}\,\omega$$

$$\psi_2(z,t) = \underbrace{T \cdot A \cos(\omega t - k_2 z)}_{\text{prošlá}} \qquad k_2 = \sqrt{\frac{\rho_2}{T_2}}\,\omega$$

1. podmínka: $\psi_1(0,t) = \psi_2(0,t)$:

$$A\cos(\omega t) + R\cdot A\cos(\omega t) = T\cdot A\cos(\omega t) \quad / A\cos(\omega t),\ \cos\omega t \neq 0$$

$$1 + R = T$$

2. podmínka: $T_1 \frac{\partial}{\partial z}\psi_1(0,t) = T_2 \frac{\partial}{\partial z}\psi_2(0,t)$

$$T_1(k_1 A\sin(\omega t) - k_1 R A\sin(\omega t)) = T_2 k_2 A\sin(\omega t)$$

$$T_1 k_1 - T_1 k_1 R = T_2 k_2 \cdot T_2$$

$$T_1 k_1 (1-R) = (1+R) k_2 \cdot T_2$$

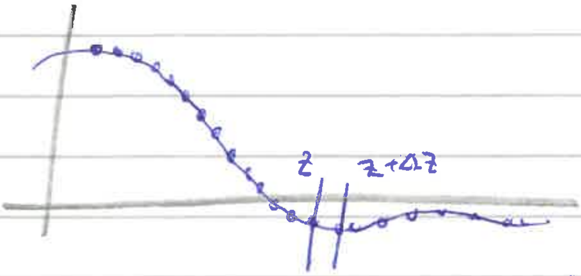
$$\sqrt{T_1\rho_1}\,\omega(1-R) = (1+R)\sqrt{T_2\rho_2}\,\omega$$

$$\sqrt{T_1\rho_1} - R\sqrt{T_1\rho_1} - \sqrt{T_2\rho_2} + R\sqrt{T_2\rho_2}$$

$$R = \frac{\sqrt{T_1\rho_1} - \sqrt{T_2\rho_2}}{\sqrt{T_1\rho_1} + \sqrt{T_2\rho_2}} = \frac{Z_1 - Z_2}{Z_1 + Z_2} \in (-1, 1)$$

$$T = R + 1 = \frac{2Z_1}{Z_1 + Z_2} \in (0, 2)$$

3.3. Energetické veličiny



↳ kinetická: $\frac{1}{2}\rho_L \Delta z \left(\frac{\partial\psi}{\partial t}\right)^2$

potenciální $U_0 + \frac{1}{2}k(\Delta y)^2 = \frac{1}{2}\frac{T}{\Delta z}\left(\frac{\partial\psi}{\partial z}\Delta z\right)^2 = \frac{1}{2}T\left(\frac{\partial\psi}{\partial z}\right)^2 \Delta z$

Na element Δz: $E = \underbrace{\left[\frac{1}{2}\rho_L\left(\frac{\partial\psi}{\partial t}\right)^2 + \frac{1}{2}T\left(\frac{\partial\psi}{\partial z}\right)^2\right]}_{\mathcal{E}}\Delta z$

hustota energie $\mathcal{E} = \frac{dE}{dz}$

$\frac{\partial\mathcal{E}}{\partial t} = -\frac{\partial S}{\partial z}$, S - hustota toku energie

$$\boxed{S = -T\frac{\partial\psi}{\partial t}\frac{\partial\psi}{\partial z}}$$

$$\underbrace{\langle|S_D|\rangle_T}_{I_D} = \underbrace{\langle|S_0|\rangle_T}_{I_0} + \underbrace{\langle|S_p|\rangle_T}_{I_p}$$

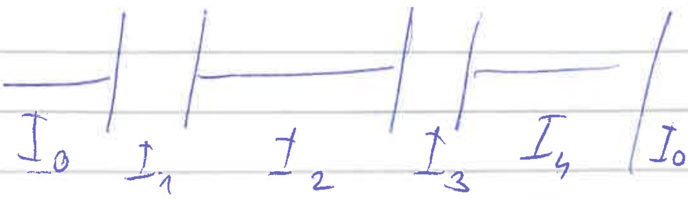
Světelná vlna (paprsek) (Maxwell)

$\omega = k\frac{c}{n}$, $v = \frac{c}{n} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\varepsilon_r\mu_r}}$ / $\mu_r \approx 1$

$Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r}{\varepsilon_r}}Z_0$

Z_0: impedance vakua $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377\,\Omega$

3.11.



$I_1 = I_0\mathcal{T}_{12} = I_0\frac{4Z_1Z_2}{(Z_1+Z_2)^2}$

$I_0 = \frac{\frac{4Z_0^2}{n_1n_2}}{\left(\frac{Z_0}{n_1}+\frac{Z_0}{n_2}\right)^2} = \frac{4n_1n_2}{(n_1+n_2)^2}$

$I = I\mathcal{T} \quad I \quad I\mathcal{T}^2$

Telegrafní rovnice (9. cvičení)

Lecherovo vedení

$$-\frac{\partial u}{\partial z}(z,t) = R\, i(z,t) + L\,\frac{\partial i}{\partial t}(z,t)$$

$$-\frac{\partial i}{\partial z}(z,t) = G\, u(z,t) + C\,\frac{\partial u}{\partial t}(z,t)$$

Předpoklady: $R = 0,\ G = 0$

$$\Rightarrow\ \frac{\partial^2 u}{\partial z^2} = LC\,\frac{\partial^2 u}{\partial t^2}\ ,\quad \frac{\partial^2 i}{\partial z^2} = LC\,\frac{\partial^2 i}{\partial t^2}$$

vlnová rovnice nedisperzní: $k^2 = LC\omega^2$

$\omega = \frac{1}{\sqrt{LC}}\,k$

$v_\varphi = v_g = \frac{1}{\sqrt{LC}}$

3. 14

Navazující podmínky: $u_2(0,t) = u_1(0,t)$, $R, T = ?$

$i_2(0,t) = i_1(0,t)$,

$u_1(z,t) = F_p(z - v_1 t) + F_o(z + v_1 t)$, $u_2 = F_p(z - v_2 t)$

$i_1(z,t) = G_p(z - v_1 t) + F_o(z + v_1 t)$, $i_2 = G_p(z - v_2 t)$

$$-\frac{\partial u_1}{\partial z} = L_1\,\frac{\partial i_1}{\partial t}$$

Určitě doma udělat znovu celé!

$$-\frac{\partial i}{\partial z} = C_1\,\frac{\partial u}{\partial t}$$

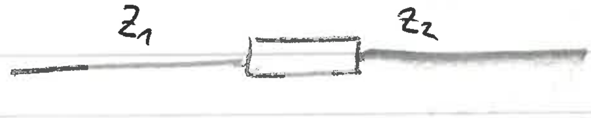
$$Z_{proud} = \sqrt{\frac{L}{C}}$$

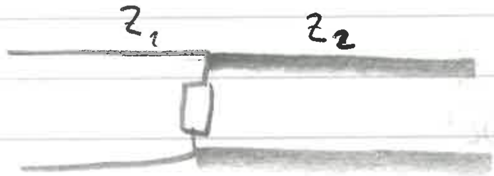
$$Z_{NAPĚTÍ} = \sqrt{\frac{C}{\ }}$$

pevný konec napětí ⟺ volný konec proudu

pevný konec proud ⟺ volný konec napětí

Příklad: $Z_1 \neq Z_2$

a) 

b) 

~~Navazující podmínky~~ :

$$u_1 = F_D(z - v_1 t) = Z_1 \cdot G_D(z - v_1 t)$$
$$i_1 = G_D(z - v_1 t)$$

$$u_2 = Z_2 G_P(z - v_2 t)$$
$$i_2 = G_P(z - vt)$$

Navazující podmínky: a) $u_2(0,t) = u_1(0,t) - R i_1(0,t)$

$i_2(0,t) = i_1(0,t)$

b) $u_2(0,t) = u_1(0,t)$

$i_1(0,t) = i_1(0,t) - \frac{1}{R} u_1(0,t)$

a) $Z_2 G_P(0 - v_2 t) = Z_1 G_D(0 - v_1 t) - R G_D(0 + v_1 t)$

$G_P(-v_2 t) = G_D(-v_1 t)$

$Z_2 G_D(-v_1 t) = Z_1 G_D(-v_1 t) - R G_D(-v_1 t)$

① $G_D = 0, \forall t, \forall z$

② $Z_2 = Z_1 - R$

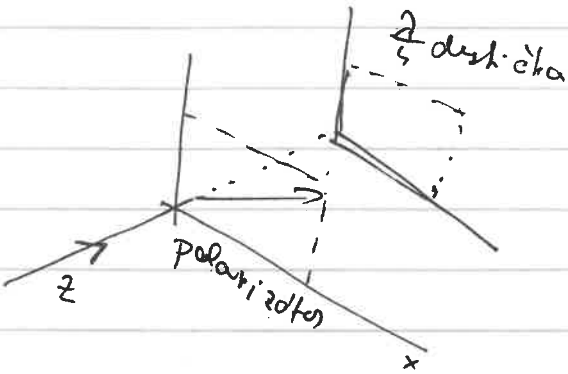
Vlny v prostoru

vlnové číslo $\longrightarrow$ vlnový vektor $\vec{k}$

$\vec{k}$ < velikost: $k = |\vec{k}| = \frac{2\pi}{\lambda}$; směr: $\frac{\vec{k}}{|\vec{k}|} = \vec{s}$, směr šíření vlny

disperzní vztah $\omega = |\vec{k}| v$

19. prosince (Středa)



a) vstup libovolný $\rightarrow$ lin. pol. světlo @ $45°$ $\rightarrow$ pravotočivé kruhové pol.

b) levotočivé kruh. pol. $\rightarrow$ lin. pol. světlo @ $45°$ $\rightarrow$ $\emptyset$

a) před destičkou:

$$\vec{E}(z,t) = E_0(\vec{x}_0 + \vec{y}_0)\cos(\omega t - kz)$$

v destičce:

E_x ... řádná osa (n_o)

E_y ... mimořádná osa (n_e)

$$E_x = E_0 \cos(\omega t - k_o z) \quad , \quad k_o = \frac{2\pi n_o}{\lambda_0}$$

$$E_y = E_0 \cos(\omega t - k_e z) \quad , \quad k_e = \frac{2\pi n_e}{\lambda_0}$$

Po průchodu tloušťkou d: E_x a E_y získají $\Delta\varphi = \frac{\pi}{2}$

Za destičkou:

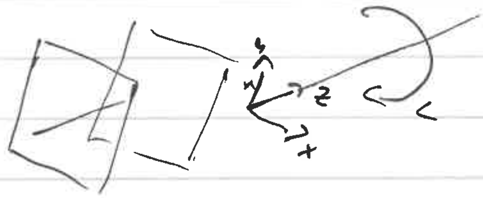
$$E_x = E_0 \cos(\omega t - kz + \varphi_0)$$

$$E_y = E_0 \cos(\omega t - kz + \varphi_0 + \frac{\pi}{2}) = -E_0 \sin(\omega t - kz + \varphi_0)$$

b) $E_x = E_0 \cos(\omega t + kz)$

$E_y = -E_0 \sin(\omega t + kz)$

$E_z = 0$



za doštičkou:

$E_x = E_0 \cos(\omega t + kz)$

$E_y = -E_0 \sin(\omega t + kz + \frac{\pi}{2}) = -E_0 \cos(\omega t + kz)$

$E_z = 0$

$\vec{E} = E_0 \cdot (\vec{x} + \vec{y}) \cos(\omega t + kz)$

Polarizátor popíšeme $\frac{1}{\sqrt{2}}(\vec{x}_0 + \vec{y}_0)$

za polarizátorom:

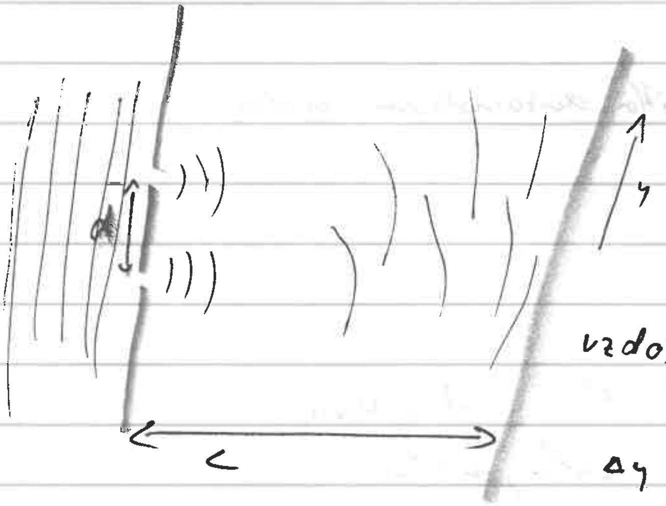
$\vec{E} = E_0 (\vec{x}_0 - \vec{y})(\vec{x} + \vec{y}) \frac{1}{\sqrt{2}} \cos(\omega t + kz) = 0$

Difrakce, interference (kap. 7)

7.6 : 2 štěrbiny : $d = 0{,}5\,mm$

$\lambda = 632{,}8\,nm$

$L = 3\,m$



$\sin \vartheta_m = m \frac{\lambda}{d} \approx \frac{y}{L}$

$y_m = m \frac{\lambda L}{d}$, poloha maxim $(0, \pm 1, \pm 2, \dots)$

vzdálenost: $\Delta y = y_{m+1} - y_m = \frac{\lambda L}{d}$

$\Delta y = \frac{\lambda L}{d} \approx 0{,}3\,mm$

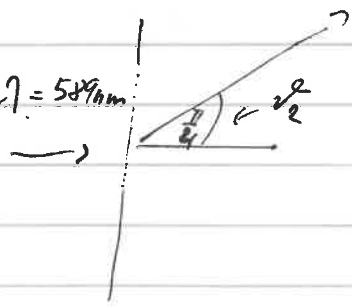
7.8

Kolik interferenčních maxim (nižšího)?

13400 vrypů na mřížce na palec

$\lambda = 0{,}55\,\mu m$

~~MA~~ 7,39: $\lambda = 589\,nm$



$\sin \vartheta_m = m \frac{\lambda}{d}$

$\frac{\sqrt{2}}{2} = 2 \frac{\lambda}{d} \Leftrightarrow d = \frac{4\lambda}{\sqrt{2}} \approx 1600\,nm$

7,42:

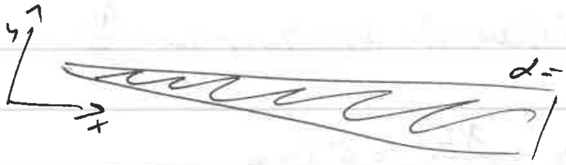
Kolik vrypů musí mít mřížka, aby rozlišila spektrální čáry $\lambda_1 = 589\,nm$, $\lambda_2 = 589{,}6\,nm$

Úhlové rozlišení oka: $3 \cdot 10^{-4}$ rad

7.12:

Jak daleko rozlišíme světla automobilu ($d=1m$)

7.25:

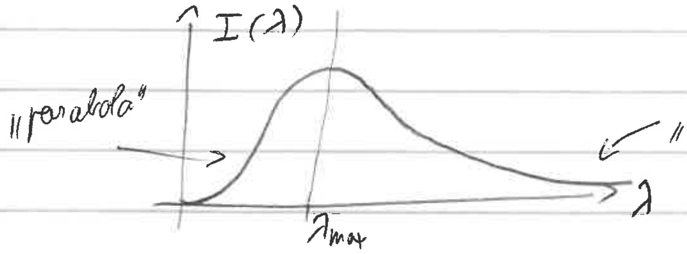


$\alpha = 0{,}1°$ $\lambda = 500\,nm$

sklo, $n = 1{,}5$

Základy atomové fyziky

• Záření A.Č.T



Celková intenzita: $I = \sigma T^4$ $\left[\frac{W}{m^2}\right]$ (Stefan-Boltzmann)

↑ S-B konstanta

$\sigma \approx 5{,}67 \cdot 10^{-8} \frac{W}{m^2 K^4}$

Poloha nejvíce zastoupené vln. délky: $\lambda_{max} = \frac{b}{T}$ (Wien)

Wienova konstanta

$b \approx 2{,}898\ mm \cdot K$

Fotoefekt

$E_{foton} = \hbar \omega$

$E_{elektron\,(vázaný)} = -W$

$E_{elektron\,(volný)} = \frac{1}{2} m_e v^2$

$$\hbar \omega - W = \frac{1}{2} m_e v^2$$

• de Broglieho vlna

vlna popisující částici

$\vec{k} := \frac{\vec{p}}{\hbar}$, $\omega := \frac{E}{\hbar}$

Určete zákl. a 1. excitovaný stav 1D elektronu na intervalu délky $L = 4 \cdot 10^{-10}$ m

$e^{i\varphi}$ $\quad e\, i e^{i\varphi}$

$\cancel{\int_0^{2\pi} e(\cos x + i \sin x)\, dx = e[\sin x]_0^{2\pi} + i[\cos x]_0^{2\pi}}$