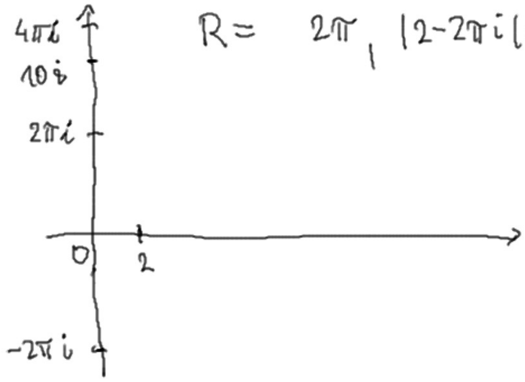
$0, 2, 10i$



$R = 2\pi, \ |2-2\pi i| = 2\sqrt{1+\pi^2}, \ 4\pi - 10$

(22) $f(z) = \dfrac{g(z)}{P(z)}, \ g \in H(\mathbb{C}), \ P(z) = (z-a_1)(z-a_2)\cdots(z-a_n), \ n \in \mathbb{N}$

$a_1, \dots, a_n \in \mathbb{C}$, navzájem různé

$\Omega = \mathbb{C}$, γ reg. uzavřená křivka v $\mathbb{C}$, $a_1, \dots, a_n \notin \langle\gamma\rangle$

$\int_\gamma f = ?$

Řešení. $\dfrac{1}{(z-a_1)\cdots(z-a_n)} = \sum_{k=1}^{n} \dfrac{A_k}{z-a_k}, \ A_k \in \mathbb{C} \ \Big/ \begin{array}{l} 1 \le j \le n \\ (z-a_j), z \to a_j \end{array}$

$$A_j = \frac{1}{\prod\limits_{\substack{1 \le k \le n \\ k \ne j}} (a_j - a_k)} = \frac{1}{P'(a_j)}$$

$$P'(z) = \sum_{k=1}^{n} \prod_{\substack{1 \le l \le n \\ l \ne k}} (z - a_l), \quad P'(a_j) = \prod_{\substack{1 \le l \le n \\ l \ne j}} (a_j - a_l)$$

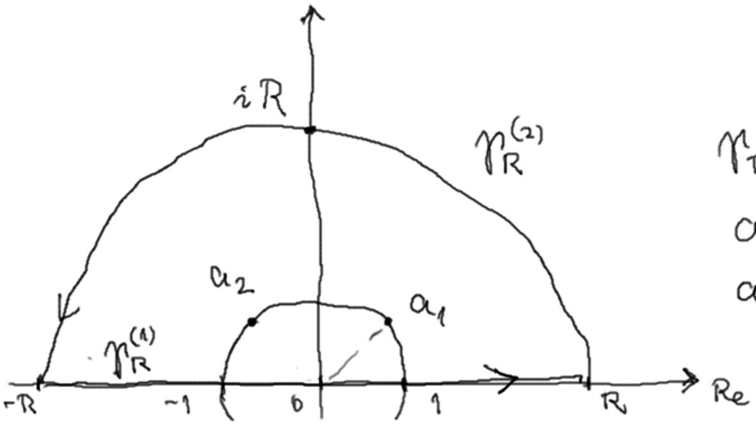
$$\int_\gamma f = \sum_{k=1}^{n} A_k \int_\gamma \frac{g(z)}{z-a_k}\,dz = 2\pi i \sum_{k=1}^{n} \underbrace{\frac{g(a_k)}{P'(a_k)}}_{\operatorname{res}_{a_k} f} \operatorname{ind}_\gamma(a_k)$$

(23) $$A := \int_{-\infty}^{\infty} \frac{x^2}{x^4+1}\,dx = \lim_{R\to\infty} \int_{-R}^{R} \frac{x^2}{x^4+1}\,dx = ?$$

Řešení $f(z) := \dfrac{z^2}{z^4+1} = \dfrac{g(z)}{P(z)}$ $\quad g(z) = z^2, \ P(z) = z^4+1$

$= (z-a_1)(z-a_2)(z-a_3)(z-a_4)$

$z^4 = -1 = e^{i\pi}$: $a_1 = e^{i\pi/4}, \ a_2 = e^{i3\pi/4}, \ a_3 = e^{i5\pi/4} = e^{-i3\pi/4}, \ a_4 = e^{i7\pi/4} = e^{-i\pi/4}$



$\gamma_R = \gamma_R^{(1)} + \gamma_R^{(2)}$

$a_1, a_2 \in \operatorname{int}(\gamma_R)$

$a_3, a_4 \in \operatorname{ext}(\gamma_R)$

$\operatorname{ind}_{\gamma_R}(a_1) = \operatorname{ind}_{\gamma_R}(a_2) = 1$

$\operatorname{ind}_{\gamma_R}(a_3) = \operatorname{ind}_{\gamma_R}(a_4) = 0$