

$$\int_{\mathbb{P}_R} f = 2\pi i \sum_{k=1}^4 \operatorname{res}_{a_k} f \cdot \operatorname{ind}_{\mathbb{P}_R}(a_k) = 2\pi i (\operatorname{res}_{a_1} f + \operatorname{res}_{a_2} f)$$

$$\operatorname{res}_{a_1} f = \frac{g(a_1)}{(a_1 - a_2)(a_1 - a_3)(a_1 - a_4)} = \frac{g(a_1)}{P'(a_1)} = \frac{a_1^2}{4a_1^3} = \frac{1}{4a_1}$$

$$\operatorname{res}_{a_2} f = \frac{g(a_2)}{(a_2 - a_1)(a_2 - a_3)(a_2 - a_4)} = \frac{g(a_2)}{P'(a_2)} = \frac{a_2^2}{4a_2^3} = \frac{1}{4a_2}$$

$$\begin{aligned} \int_{\mathbb{P}_R} f &= \frac{2\pi i}{4} (\bar{a}_1 + \bar{a}_2) = \frac{2\pi i}{4} (e^{i7\pi/4} + e^{i5\pi/4}) = \frac{2\pi i}{4} e^{i3\pi/2} (e^{i\pi/4} + e^{-i\pi/4}) \\ &= -\frac{2\pi i}{4} i 2 \underbrace{\cos \frac{\pi}{4}}_{\frac{1}{\sqrt{2}}} = \frac{2\sqrt{2}\pi}{4} = \frac{\pi}{\sqrt{2}} \end{aligned}$$

$$\mathbb{P}_R^{(1)}(t) = t, \quad t \in [-R, R], \quad \int_{\mathbb{P}_R} f = \int_{-R}^R \frac{t^2}{t^4 + 1} dt$$

$$\mathbb{P}_R^{(2)}(t) = R e^{it}, \quad t \in [0, \pi]$$

$$\left| \int_{\mathbb{P}_R^{(2)}} f \right| = \left| \int_0^\pi i R e^{it} \frac{R^2 e^{2it}}{R^4 e^{4it} + 1} dt \right| \leq \int_0^\pi \frac{R^3}{|R^4 e^{4it} + 1|} dt$$

$$\leq \int_0^\pi \frac{R^3}{R^4 - 1} dt = \frac{\pi R^3}{R^4 - 1} \xrightarrow{|a+b| \geq |a| - |b|} 0, \quad R \rightarrow \infty$$

$$\lim_{R \rightarrow \infty} \int_{\mathbb{P}_R} f = \int_{-\infty}^{\infty} \frac{t^2}{t^4 + 1} dt = A$$

$$\forall R > 1, \quad \int_{\mathbb{P}_R} f = \frac{\pi}{\sqrt{2}}$$

$$A = \frac{\pi}{\sqrt{2}}$$