(31) $\ln \in H(\Omega)$, $\Omega := \mathbb{C} \setminus (-\infty, 0]$

$f(z) := \dfrac{\ln(1+z)}{z}$, $f \in H(\tilde{\Omega})$, $\tilde{\Omega} := \mathbb{C} \setminus \left((-\infty, -1] \cup \{0\}\right)$

$\operatorname{res}_0(f) = ?$ samostatně

(32) $a \in \mathbb{C}$; $f(z) := \exp\left(\frac{a}{z}\right)$, $\forall z \in \mathbb{C} \setminus \{0\}$

$\gamma(t) := re^{it}$, $t \in [0, 2\pi]$, $r > 0$

$\int_\gamma f = ?$ samostatně

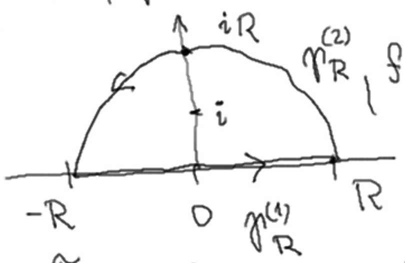
(33) $x \in \mathbb{R}$, $z \in \mathbb{C} \setminus [0, +\infty)$, $\displaystyle\int_{-\infty}^{\infty} \frac{e^{ixk}}{k^2 - z}\, dk = ?$

Řešení. výpočet pro $z < 0$, subst. $z = -\mu^2$, $\mu > 0$

$$\int_{-\infty}^{\infty} \frac{e^{ixk}}{k^2 + \mu^2}\, dk \underset{k = \mu k'}{=} \frac{1}{\mu} \int_{-\infty}^{\infty} \frac{e^{i\mu x k}}{k^2 + 1}\, dk \text{ ; položíme } \mu x =: y$$

$$\int_{-\infty}^{\infty} \frac{e^{iyk}}{k^2 + 1}\, dk, \ y \in \mathbb{R}$$

$y > 0$:



$\gamma_R^{(2)}$, $f(z) := \dfrac{e^{iyz}}{z^2 + 1}$, $f \in H(\mathbb{C} \setminus \{-i, +i\})$

$\gamma_R^{(2)}(t) = Re^{it}$, $t \in [0, \pi]$

γ_R = celá křivka

$$\int_{\gamma_R^{(2)}} f = \int_0^{\pi} \frac{e^{iy(R\cos t + iR\sin t)}}{R^2 e^{2it} + 1}\, iRe^{it}\, dt \longrightarrow 0 \text{ pro } R \to +\infty$$

(samostatně)

$$\int_{\gamma_R^{(1)}} f \longrightarrow \int_{-\infty}^{\infty} \frac{e^{iyk}}{k^2 + 1}\, dk \text{ pro } R \to +\infty$$

$$\int_{\gamma_R} f = 2\pi i \operatorname{res}_i(f) = 2\pi i \frac{e^{-y}}{2i} = \pi e^{-y}$$

$$\forall y \in \mathbb{R}, \quad \int_{-\infty}^{\infty} \frac{e^{iyk}}{k^2 + 1}\, dk = \pi e^{-|y|}$$

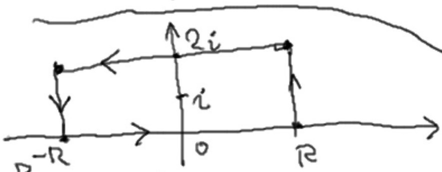
$$\mu > 0, \quad \int_{-\infty}^{\infty} \frac{e^{ixk}}{k^2 + \mu^2}\, dk = \frac{\pi}{\mu} e^{-\mu|x|}$$

$$\forall z \in \mathbb{C} \setminus [0, +\infty), \quad \int_{-\infty}^{\infty} \frac{e^{ixk}}{k^2 - z}\, dk = \frac{\pi}{\sqrt{-z}} e^{-\sqrt{-z}\,|x|} \quad \text{samostatně}$$

[$s(z) := \sqrt{z}$ def. na $\mathbb{C} \setminus (-\infty, 0]$; $z \in \mathbb{C} \setminus [0, +\infty) \Rightarrow -z \in \mathbb{C} \setminus (-\infty, 0]$

$\sqrt{-z} = s(-z)$]

(34) $\displaystyle\int_{\gamma_R} \frac{dz}{z - i}$, $R \to +\infty$, γ_R:



(podrobnosti samostatně)

Řešení.

$$2\pi i = \int_{\gamma_R} \frac{dz}{z - i} = \lim_{R \to \infty} \left(\int_{-R}^{R} \frac{dt}{t - i} - \int_{-R}^{R} \frac{dt}{t + i} \right) = 2i \int_{-\infty}^{\infty} \frac{dt}{t^2 + 1} \Rightarrow \int_{-\infty}^{\infty} \frac{dt}{t^2 + 1} = \pi$$