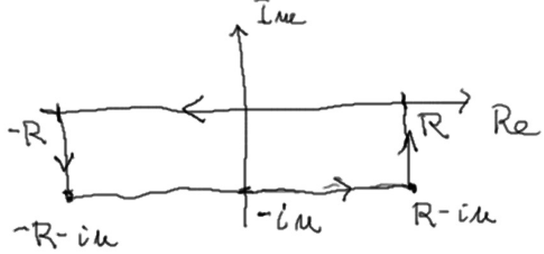
(35) $a>0,\ x\in\mathbb{R},\ \int_{-\infty}^{\infty} e^{ixy}e^{-ay^2}\,dy = ?$

Řešení. $$\int_{-\infty}^{\infty} e^{ixy}e^{-ay^2}\,dy \underset{\sqrt{a}y=t}{=} \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} e^{i\frac{x}{\sqrt{a}}t-t^2}\,dt \underset{\frac{x}{\sqrt{a}}=2u}{=} \frac{1}{\sqrt{a}}e^{-u^2}\underbrace{\int_{-\infty}^{\infty} e^{-(t-iu)^2}\,dt}$$

$f(z)=e^{-z^2}$,



γ_R

$$0=\int_{\gamma_R} f\ ,\ R\to+\infty \qquad \int_{-\infty}^{\infty} e^{-(t-iu)^2}\,dt = \int_{-\infty}^{\infty} e^{-t^2}\,dt = \sqrt{\pi}$$

(podrobnosti samostatně)

$$\int_{-\infty}^{\infty} e^{ixy}e^{-ay^2}\,dy = \sqrt{\frac{\pi}{a}}\,e^{-\frac{x^2}{4a}}$$

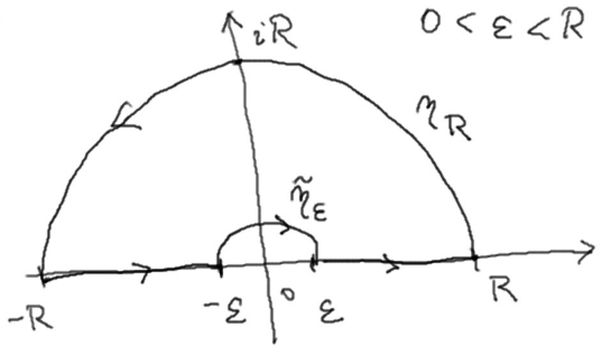
(36) $\int_0^{\infty}\frac{\sin x}{x}\,dx = ?$

Řešení. $$\int_0^{\infty}\frac{\sin x}{x}\,dx = \frac{1}{2}\int_{-\infty}^{\infty}\frac{\sin x}{x}\,dx$$

$0<\varepsilon<R$

$f(z):=\frac{e^{iz}}{z}$, $f\in H(\mathbb{C}\setminus\{0\})$

$\gamma_{\varepsilon,R}$:



$$0=\int_{\gamma_{\varepsilon,R}} f$$

$R\to+\infty$: $\int_{\eta_R} f \longrightarrow 0$ samostatně

$\eta_\varepsilon = -\tilde{\eta}_\varepsilon$

$\eta_\varepsilon(t) = \varepsilon e^{it},\ t\in[0,\pi]$

$$0=\lim_{R\to\infty}\underbrace{\left(\int_{-R}^{-\varepsilon}\frac{e^{ix}}{x}\,dx+\int_{\varepsilon}^{R}\frac{e^{ix}}{x}\,dx\right)}_{\int_\varepsilon^R \frac{e^{ix}-e^{-ix}}{x}\,dx \,=\, 2i\int_\varepsilon^R\frac{\sin x}{x}\,dx} + \int_{\tilde{\eta}_\varepsilon} f$$

$$2i\int_{\varepsilon}^{\infty}\frac{\sin x}{x}\,dx = \int_{\eta_\varepsilon} f = \int_0^{\pi}\frac{e^{i\varepsilon(\cos t+i\sin t)}}{\cancel{\varepsilon e^{it}}}\,i\cancel{\varepsilon e^{it}}\,dt\ ,\quad \varepsilon\to0+$$

$$2\int_0^{\infty}\frac{\sin x}{x}\,dx = \pi\ ,\qquad \int_0^{\infty}\frac{\sin x}{x}\,dx = \frac{\pi}{2}$$

(podrobnosti samostatně)