Váchа 1

① $M = (0,\infty)\times\mathbb{R}$ x, y

$$X = \frac{1}{x^2}\frac{\partial}{\partial x} + \frac{y}{x^3}\frac{\partial}{\partial y}$$ je X úplné?

$\varphi = ?$

$\dot{x} = \frac{1}{x^2} \rightarrow x^2 \cdot \dot{x} = 1 /\int \rightarrow \int x^2\,dx = \int 1\,dt$

$\frac{1}{3}x^3 = t + c_1$

$x^3 = 3t + c_2$

$x = \sqrt[3]{3t + c_2}$

$\dot{y} = \frac{y}{x^3}$

$\frac{\dot{y}}{y} = \frac{1}{x^3}$

$\frac{\dot{y}}{y} = \frac{1}{3t + c} \quad /\int$

$3t + c = \tau$

$3\,dt = d\tau$

$\ln(x) + \ln y = \ln(xy)$

$\ln y = \int \frac{1}{3t + c_2}\,dt = \frac{1}{3}\ln(3t + c_2) + D_1 = \frac{1}{3}\ln(3\frac{1}{3}t + c_2 D)$

$y = (D3t + c)^{1/3} = \sqrt[3]{3tD + c} = \sqrt[3]{3Dt + Dc}$

$y(0) = \sqrt[3]{x_0^3 D} = y_0$

$x_0\sqrt[3]{D} = y_0$

$\lim_{t\to 0^+} x(t) = x_0$

$y(0) = y_0$

$x(t) = \sqrt[3]{3t + x_0^3}$

$y(t) = \sqrt[3]{3\left(\frac{y_0}{x_0}\right)^3 t + y_0^3}$

$$\varphi^t_{(x_0,y_0)} = \begin{pmatrix} \sqrt[3]{3t + x_0^3} \\ \sqrt[3]{3\left(\frac{y_0}{x_0}\right)^3 t + y_0^3} \end{pmatrix}$$

~~X je úplné [illegible] M je definován všude~~

X není úplné, $t < 0$; $\varphi^t_x < 0 \notin M$

Vaiča 2

(2) $M = \mathbb{R}^2[x,y] \wedge N = \mathbb{R}^2[u,v]$

$\phi(X) = X^i \frac{\partial \varphi^a}{\partial x^i} \frac{\partial}{\partial y^a}\Big|_{\phi(p)}$

$\phi : M \to N \qquad u(\phi(x,y)) = xy$

$v(\phi(x,y)) = y^2$

$X = y\,\partial_x + 2y\,\partial_y = y\frac{\partial}{\partial x} + 2y\frac{\partial}{\partial y} \Big| \mathcal{H} = (v^3 - u)\, du \wedge dv$

$\phi_*(X|_p) = \phi_*\left(y\frac{\partial}{\partial x} + 2y\frac{\partial}{\partial y}\right) = y \cdot y \cdot \frac{\partial}{\partial u}\Big|_p + \cancel{\ } \cdot 2y \frac{\partial}{\partial v}\Big|_p$

$\frac{\partial}{\partial x} = \frac{\partial u}{\partial x}\frac{\partial}{\partial u} + \frac{\partial v}{\partial x}\frac{\partial}{\partial v} \qquad \frac{\partial(xy)}{\partial y} = \frac{\partial u}{\partial x}$

$+ \underset{\frac{\partial v}{\partial x}}{y \cdot 0} + \underset{\frac{\partial u}{\partial y}}{x} \cdot 2y \frac{\partial}{\partial u}\Big|_p = (2xy + y^2)\frac{\partial}{\partial u}\Big|_p + 4y^2 \frac{\partial}{\partial v}\Big|_p =$

$u, v \in \mathbb{R}$

$= (2u + v)\frac{\partial}{\partial u}\Big|_{(xy,y^2)} + 4v\frac{\partial}{\partial v}\Big|_{(xy,y^2)}$

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ako je definovaná všade na $\mathbb{R}^2[u,v]$

$\phi^*(\mathcal{H}) = \phi^*((v^2 - u)\, du \wedge dv) =$

$= (y^4 - xy)\left[\underbrace{(y\,dx + x\,dy)}_{\phi^*(du)} \wedge \underbrace{(2y\,dy)}_{\phi^*(dv)}\right] = \underline{(y^4 - xy)\, 2y^2\, dx \wedge dy} =$

0

$= \underline{(2y^6 - 2xy^3)\,dx \wedge dy}$

Vácha 3

(3) $M = \mathbb{R}^2[x,y]$

$\alpha = x^2 y\, dx - x^3 dy$

$$d\alpha = x^2\, dy \wedge dx - 3x^2\, dx \wedge dy = (x^2 - 3x^2)\, dx \wedge dy = -4x^2 (dx \wedge dy)$$

$= -4x^2\, dx \wedge dy$

$Y = x \frac{\partial}{\partial y}$

$$\mathcal{L}_Y \alpha = \mathcal{L}_Y (x^2 y\, dx - x^3 dy) = \mathcal{L}_Y (x^2 y)\, dx + x^2 y\, \mathcal{L}_Y (dx) - \mathcal{L}_Y (x^3)\, dy - x^3 \mathcal{L}_Y (dy) =$$

$$= x \frac{\partial x^2 y}{\partial y} dx + x^2 y\, d0 - x \underbrace{\frac{\partial x^3}{\partial y}}_{=0} dy - x^3 d(x \underbrace{\frac{\partial y}{\partial y}}_{=1}) =$$

$= x^3\, dx - x^3\, dx = 0$

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(4) $N = (-\frac{\pi}{2}; \frac{\pi}{2})^{\times 4}$ p, q, r, s

$$\omega = \cos p \, dp \wedge dq - dq \wedge ds + dr \wedge ds - \cos p \, dp \wedge dr$$

ω syplektická forma na N? a) nedegenerovaná?
b) uzavřená?

$\omega(X, Y) = ?$

b) $d\omega = 0$ ✓ zřejmé: $\frac{\partial \omega_i}{\partial \omega_j} = 0$

$dp = e^1_q \equiv e^{\alpha\pi}_1$
$dq = e^2$
$\vdots$

$$X_j = X_i \frac{\partial}{\partial x_i} = P_j \frac{\partial}{\partial p} + Q_j \frac{\partial}{\partial q} + R_j \frac{\partial}{\partial r} + S_j \frac{\partial}{\partial s}$$

$$\omega(X_1, X_2) = \cos p \, P_1 Q_2 - Q_1 S_2 + R_1 S_2 - \cos p \, P_1 R_2 =$$

$$R_1 S_2 = Q_1 S_2$$

$$P_1 Q_2 = P_1 R_2$$

$$R_1 = Q_1$$

$$\wedge \; Q_2 = R_2$$

$$= \cos p \, P_1 (Q_2 - R_2) + S_2 (R_1 - Q_1)$$

$\forall p$

$$\cos p \; P_1 (Q_2 - R_2) = S_2 (R_1 - Q_1)$$

$\forall P_1, Q_1, R_1 S_1 \neq 0$

$$S_2 = \frac{\cos p \, P_1 (Q_2 - R_2)}{R_1 - Q_1}$$, lze nenaplnit.

$X_i \neq 0 \Leftrightarrow i_X \omega \neq 0$

$\qquad = X_i^p dx^i - X_x^i dp_i$

$\to$ je nedegenerovaná

$$\omega = dp^i \wedge dx^i$$

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Vácha 5

$\cancel{\omega = da \wedge db + dc \wedge de}$

$dp - dq =$

$p - q = a$

$da = d(p-q) = dp - dq$

$dr \wedge ds - dq \wedge ds = \underbrace{(dr - dq)}_{da} \wedge ds$

$\cos p \cdot (dp \wedge dq - dp \wedge dr) = \cos p \, dp (\underbrace{dq - dr}_{-da})$

$\omega = \cos p \, da \wedge dp + da \wedge ds$

$\cancel{b(p,s) = ?}$

$\omega = da \wedge d\pi + da \wedge ds$

$\pi = \sin p$

$d\pi = \cos p \, dp$

$\omega = \cos p \, dp (dq \wedge dr) + (-dq + dr) \wedge ds$

$= \cos p \, dp (dq \wedge dr) + ds (dq - dr) = (\cos p \, dp - ds) \wedge (dq - dr)$

$\pi = \sin p - s$ | $d\pi = \cos p \, dp - ds$

$\rho = q - r$ | $d\rho = dq - dr$

$$\boxed{\omega = d\pi \wedge d\rho}$$

$\cancel{X \neq 0 \iff X_i^p dx^i - X_x^j dp_i \neq 0}$

$\cancel{X_x^i \frac{\partial}{\partial x^i} + X_j^p \frac{\partial}{\partial p_j}}$ $\lambda = p_i dx^i$

$$\boxed{\lambda = \pi \, d\rho}$$

$$\boxed{\omega = d\lambda}$$ – ω je symplektická