RMF úkol z 25.9.

Lukáš Vácha

27. září 2020

1 Věty o záměně v Lebegueově integrálu

Věta 1 (Levi)**.** Buď $\{\varphi_n\}_{n=1}^{\infty} \subset \Lambda$, $\varphi_n \gtrsim 0$, $\varphi \sim \sum_{n=1}^{\infty} \varphi_n$. Pak $\varphi \in \Lambda$ a $\mathbf{I}\varphi = \sum_{n=1}^{\infty} \mathbf{I}\varphi_n$.

Věta 2 (Lebesgue)**.** Buď $\{\varphi_n\}_{n=1}^{\infty} \subset \mathcal{L}$, $\varphi_n \to \varphi$ a $(\exists \varphi_0 \in \mathcal{L})(\forall n \in \mathbb{N})(|\varphi_n| \lesssim \varphi_0)$. Pak $\varphi \in \mathcal{L}$ a

$$\mathbf{I}\varphi = \lim_{n\to\infty} \mathbf{I}\varphi_n.$$

Posloupnost integrabilních funkcí je integrabilní, jestliže existuje integrabilní majoranta.

Věta 3 (o limitě)**.** Buď $M \subset \mathbb{R}^n$, (P, ρ) metrický prostor, $A \subset P$, $\alpha_0 \in A'$, $f : M \times A \mapsto \mathbb{R}$ a nechť platí:

1. Pro skoro všechna $x \in M$
$$\lim_{\substack{\alpha\to\alpha_0 \\ \alpha\in A}} f(x, \alpha) = \varphi(x),$$
2. $(\forall \alpha \in A \setminus \{\alpha_0\})(f(\ , \alpha)$ je měřitelná na $M)$,
3. $(\exists g \in \mathcal{L}(M))$(pro skoro všechna $x \in M)(\forall \alpha \in A \setminus \alpha_0)(|f(x, \alpha)| \leq g(x))$.

Potom

1. $\varphi \in \mathcal{L}(M)$,
2. $$\int_M \varphi = \lim_{\substack{\alpha\to\alpha_0 \\ \alpha\in A}} \int_M f(x, \alpha).$$

Věta 4 (o derivaci)**.** Buď M měřitelná množina, $M \subset \mathbb{R}^n$ a nechť $\mathcal{I} = \mathcal{I}^\circ \subset \mathbb{R}$. Nechť $f : M \times \mathcal{I} \mapsto \mathbb{R}$ je reálná funkce a platí:

1. Existuje $\alpha_0 \in \mathcal{I}$ takové, že $f(\ , \alpha_0) \in \mathcal{L}(M)$,
2. pro každé $\alpha \in \mathcal{I}$ platí, že $f(\ , \alpha)$ je měřitelná na M,

3. je-li $N \subset M$, $\mu(N) = 0$, pak $f(x, \ )$ je diferencovatelná na $\mathcal{I}$ pro každé $x \in M \setminus N$,

4. existuje $g \in \mathcal{L}(M)$ tak, že

$$(\forall x \in M \setminus N)(\forall \alpha \in \mathcal{I}) \left(\left| \frac{\partial f(x,\alpha)}{\partial \alpha} \right| \leq g(x) \right).$$

Potom

1. $f(\ ,\alpha) \in \mathcal{L}(M)$ pro každé $\alpha \in \mathcal{I}$,

2. $\frac{\partial}{\partial \alpha} f(\ ,\alpha) \in \mathcal{L}(M)$ pro každé $\alpha \in \mathcal{I}$,

3. a platí

$$\frac{\mathrm{d}}{\mathrm{d}\alpha} \int_M f(x,\alpha)\,\mathrm{d}x = \int_M \frac{\partial}{\partial \alpha} f(x,\alpha)\,\mathrm{d}x.$$

2 Skalární součin

Věta 5.

$$(f,g) = \int_a^b f(x)\overline{g(x)}dx$$

je skalární součin na $C[a,b] \times C[a,b]$

Důkaz. 1. linearita:

$$(\alpha f + h, g) = \int_a^b (\alpha f(x) + h(x))\overline{g(x)}dx = \int_a^b (\alpha f(x)\overline{g(x)} + h(x)\overline{g(x)})dx =$$

$$\alpha \int_a^b f(x)\overline{g(x)}dx + \int_a^b h(x)\overline{g(x)})dx = \alpha(f,g) + (h,g) \quad (1)$$

2. hermitovskost:

$$(f,g) = \int_a^b f(x)\overline{g(x)}dx = \int_a^b \overline{g(x)}f(x)dx = \int_a^b \overline{g(x)\overline{f(x)}} = \overline{\int_a^b g(x)\overline{f(x)}dx} = \overline{(g,f)} \quad (2)$$

3. Pozitivní definitnost:

$$f^2(x) \geq 0 \Rightarrow \int_a^b f^2(x)dx \geq 0 \quad (3)$$

$$\int_a^b 0dx = 0 \quad (4)$$

sporem:

$$\int_a^b f^2(x)dx = 0 \wedge (\exists x_0)(f(x_0) \neq 0) \Rightarrow (\exists U_{x_0} \subset [a,b])(\forall x \in U_{x_0})(f(x) \neq 0) \Rightarrow$$

$$\int_a^b f^2(x)dx = \int_{U_{x_0}} f^2(x)dx + \int_{[a,b]\setminus U_{x_0}} f^2(x)dx \neq 0 \quad (5)$$

, což je spor

□

3 spojitost f

Věta 6. $g \in C(\mathbb{R}^2); A \subset \mathbb{R}$ omezená, pak
$f(x) = \int_A g(x,y)dy$ je spojitá na $\mathbb{R}$

1. *Důkaz.* $(\forall (x,y) \in \mathbb{R}^2)(\forall \varepsilon_0 > 0)(\exists \delta_0 > 0)(|g(x+\delta,y) - g(x,y)| < \varepsilon_0)$
$|f(x+\delta) - f(x)| = \left|\int_A g(x+\delta,y)dy - \int_A g(x,y)dy\right| \leq \left|\int_A |g(x+\delta,y) - g(x,y)dy|\right| \leq$
$\left|\int_A \varepsilon_0\right| = \varepsilon_0 \mu(A) < \varepsilon$ □

2. *Důkaz.* g je spojtá, na $\bar{A}$ nabývá svého maxima M, g má integrabilní majorantu M:

$$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} \int_A g(x,y)dy = \int_A \lim_{x \to x_0} g(x,y)dy = \int_A g(x_0,y)dy = f(x_0) \quad (6)$$

□

4 derivace fce

1. $a_0 = 0, f(,a_0) \in \mathcal{L}, \int_0^\infty cos(-\pi x)dx$ existuje
2. $(\forall \varepsilon > 0)(\forall a \in (\varepsilon, +\infty))$

$$\left|-\int_0^\infty x^2 e^{-ax^2} cos((a-\pi)x) - xe^{-ax^2} sin((a-\pi)x)\right| \leq \left|(-x^2 - x)e^{-ax^2}\right| \leq \left|(-x^2 - x)e^{-\varepsilon x^2}\right| \quad (7)$$

, což je integrabilní majoranta

$\forall a \in (\varepsilon, \infty)$

$$f'(a) = \frac{d}{da}\int_0^\infty e^{-ax^2} cos((a-\pi)x)dx = \int_0^\infty \frac{\partial}{\partial a} e^{-ax^2} cos((a-\pi)x)dx =$$

$$-\int_0^\infty x^2 e^{-ax^2} cos((a-\pi)x) - xe^{-ax^2} sin((a-\pi)x)dx \quad (8)$$

RMF úkol do 9.10.

Lukáš Vácha

4. října 2020

1 příklad 2

$f \in \mathscr{D}'$?

a. $(f, \varphi) = \varphi(\pi) = (\delta_\pi, \varphi)$
linearita: $(f, a\varphi + \psi) = (a\varphi + \psi)(\pi) = a\varphi(\pi) + \psi(\pi) = a(f, \varphi) + (f, \psi)$
spojitost: $\varphi_n \xrightarrow{\mathscr{D}} \varphi \Rightarrow \varphi_n \rightrightarrows \varphi \Rightarrow \lim_{n\to+\infty} \varphi_n = \varphi \Rightarrow \lim(f, \varphi_n) = \lim \varphi_n(\pi) = \varphi(\pi) = (f, \varphi)$
$f \in \mathscr{D}'$

b. $(f, \varphi) = (\varphi(0))^2$
linearita: $(f, a\varphi + \psi) = [(a\varphi + \psi)(0)]^2 = (a\varphi(0) + \psi(0))^2 = a^2\varphi^2(0) + 2a\varphi(0)\psi(0) + \psi^2(0) \neq a(f, \varphi) + (f, \psi)$
$f \notin \mathscr{D}'$

2 příklad 3

$f \in \mathscr{D}'$?

a. $(f, \varphi) = \varphi^{(n)}(\pi)$
linearita: $(f, a\varphi + \psi) = (a\varphi + \psi)^{(n)}(\pi) = a\varphi^{(n)}(\pi) + \psi^{(n)}(\pi) = a(f, \varphi) + (f, \psi)$
spojitost: $\varphi_n \xrightarrow{\mathscr{D}} \varphi \Rightarrow \forall \alpha D^\alpha \varphi_n \rightrightarrows D^\alpha \varphi \Rightarrow \lim_{k\to+\infty} \varphi_k^{(n)} = \varphi^{(n)} \Rightarrow \lim(f, \varphi_k) = \lim \varphi_k^{(n)}(\pi) = \varphi^{(n)}(\pi) = (f, \varphi)$
$f \in \mathscr{D}'$

b. $(f, \varphi) = \lim_{x\to+\infty} \varphi(x) = 0 = (0, \varphi)$
$\mathscr{D}'$ je vektorový prostor $\Rightarrow 0 \in \mathscr{D}'$
$f \in \mathscr{D}'$

RMF úkol č. 3

Lukáš Vácha

13. října 2020

1 př č. 1

ad 1. $\delta(2x) \stackrel{?}{=} \frac{1}{2}\delta(x)$
$(\delta(2x); \varphi(x)) = \left[\ y = 2x\ \right] = \frac{1}{2}(\delta(y); \varphi(\frac{1}{2}y)) = \frac{1}{2}\varphi(\frac{1}{2}0) = \frac{1}{2}\varphi(0) = \frac{1}{2}(\delta(x); \varphi(x)) =$
$(\frac{1}{2}\delta(x); \varphi(x)); \forall\varphi \in \mathscr{D}$
$\delta(2x) = \frac{1}{2}\delta(x)$

ad 2. $\delta(2x) \stackrel{?}{=} 2\delta(x)$
$(LS, \varphi) = (\delta(2x); \varphi(x)) = ... = \frac{1}{2}\varphi(0)$
$(PS, \varphi) = (2\delta(x); \varphi(x)) = 2(\delta(x); \varphi(x)) = 2\varphi(0)$
Pro $\varphi \in \mathscr{D}$ tak, že $\varphi(0) = 1 : \frac{1}{2}\varphi(0) = \frac{1}{2} \neq 2 = 2\varphi(0)$
Taková φ testovací určitě existuje a rovnost pro ni neplatí.
$\delta(2x) \neq 2\delta(x)$

RMF úkol č. 3

Lukáš Vácha

17. října 2020

1 př č. 1

$\{f_n\} \subset \mathscr{D}'(\mathbb{R}^n)$ a $f \in \mathscr{D}(\mathbb{R}^n)$, pro kterou $\lim_{n\to+\infty} f_n = f$ v $\mathscr{D}'$. Doklažte, že platí: $\lim_{n\to+\infty} f_n(Ax+b) = f(Ax+b)$ v $\mathscr{D}'$ $\forall A \in \mathbb{R}^{n,n}$ regulární a $b \in \mathbb{R}^n$.

Postup

$(\lim_{n\to+\infty} f_n(Ax+b), \varphi(x)) = \lim(f_n(Ax+b), \varphi(x)) = [y = Ax+b] =$
$= \frac{1}{\det A} \lim(f_n(y), \varphi(A^{-1}(y-b))) = \frac{1}{\det A}(\lim f_n(y), \varphi(A^{-1}(y-b))) =$
$= \frac{1}{\det A}(f(y), \varphi(A^{-1}(y-b)) = (f(Ax+b), \varphi(x))$

$$\Rightarrow \lim_{n\to+\infty} f_n(Ax+b) = f(Ax+b) \tag{1}$$

RMF úkol č. 3

Lukáš Vácha

26. října 2020

1 př č. 3

Vpočítejte $\Theta(x)e^{-ax} \star \Theta(x)e^{-bx}; a, b > 0$:
$\Theta(x)e^{-ax} \star \Theta(x)e^{-bx} = \int_{-\infty}^{+\infty} \Theta(y)e^{-ay}\Theta(x-y)e^{-b(x-y)}dy = \int_{-\infty}^{+\infty} \Theta(y)e^{-ay}\Theta(x-y)e^{-bx}e^{by}dy = (\star)$
$\Theta(y)\Theta(x-y) = \begin{cases} 0; x < 0 \\ 1; x > 0 \wedge y \in (0, x) \end{cases}$
$(\star) = e^{-bx}\Theta(x)\int_{-\infty}^{+\infty} e^{-ay+by}dy = e^{-bx}\Theta(x)\int_{-\infty}^{+\infty} e^{(b-a)y}dy =$
$= \begin{cases} [a = b] = \Theta(x)e^{-bx}x = \Theta(x)xe^{-ax} \\ [a \neq b] = \Theta(x)e^{-bx}\frac{1}{b-a}(e^{(b-a)x} - 1) = \frac{\Theta(x)}{b-a}(e^{-ax} - e^{-bx}) \end{cases}$

RMF úkol č. 6

Lukáš Vácha

8. listopadu 2020

1 př č. 2

Vpočítejte $(e^{-|t|} \star e^{-|t|})(x) = ?$:

$|x-y| = \begin{cases} -(x-y); x < y \\ (x-y); x > y \end{cases}$
$x \cdot sgn(x) = |x|$

$? = (e^{-|t|} \star e^{-|t|})(x) = \int_{-\infty}^{+\infty} e^{-|x-y|} e^{-|y|} dy =$

ad 1. $[x > 0] = \int_{-\infty}^{0} e^{-(x-y)} e^{y} dy + \int_{0}^{x} e^{-(x-y)} e^{-y} dy + \int_{x}^{+\infty} e^{x-y} e^{-y} dy = e^{-x} \int_{-\infty}^{0} e^{2y} dy + e^{-x} \int_{0}^{x} dy + e^{x} \int_{x}^{+\infty} e^{-2y} dy = \frac{1}{2} e^{-x}(1-0) + x e^{-x} - \frac{1}{2} e^{x}(0 - e^{-2x}) = (x+1) e^{-x}$

ad 2. $[x < 0] = \int_{-\infty}^{x} e^{-(x-y)} e^{y} dy + \int_{x}^{0} e^{x-y} e^{y} dy + \int_{0}^{+\infty} e^{x-y} e^{-y} dy = e^{-x} \int_{-\infty}^{x} e^{2y} dy + e^{x} \int_{x}^{0} dy + e^{x} \int_{0}^{+\infty} e^{-2y} dy = \frac{1}{2} e^{-x}(e^{2x} - 0) - x e^{x} - \frac{1}{2} e^{x}(0-1) = (-x+1) e^{x}$

dohromady:

$$? = (x \cdot sgn(x) + 1) e^{-x \cdot sgn(x)} = (|x| + 1) e^{-|x|} \tag{1}$$

RMF úkol č. 7

Lukáš Vácha

11. listopadu 2020

1 př č. 2

$(\mathscr{F}_x[\delta(x,y)](\xi,y),\varphi(\xi,y)) = (\delta(x,y),\mathscr{F}_x[\varphi(\xi,y)](x,y)) = (\delta(x)\otimes\delta(y),\mathscr{F}_x[\varphi(\xi,y)](x,y)) = (\delta(y),(\delta(x),\mathscr{F}_x[\varphi(\xi,y)](x,y))) = (\delta(y),(\mathscr{F}_x[\delta(x)](\xi),\varphi(\xi,y))) = (\delta(y),(1(\xi),\varphi(\xi,y))) = (\delta(y)\otimes 1(\xi),\varphi(\xi,y))$

$$\mathscr{F}_x[\delta(x,y)](\xi,y) = \delta(y)\otimes 1(\xi) \tag{1}$$

RMF úkol č. 8

Lukáš Vácha

15. listopadu 2020

1 př č. 3

ad 1. $\mathscr{L}[\sin(ax)](\xi) = \int_0^{+\infty} e^{-x\xi}\sin(ax)dx = \frac{1}{2i}\int_0^{+\infty} e^{(ia-\xi)x} - e^{(-ia-\xi)x}dx = \frac{1}{2i}\left(\frac{1}{-ia+\xi} - \frac{1}{ia+\xi}\right) = \frac{ia+\xi-\xi+ia}{2i(\xi^2+b^2)} = \frac{a}{\xi^2+a^2}$

$$\mathscr{L}[\sin(ax)](\xi) = \frac{a}{\xi^2+a^2} \quad (1)$$

ad 2. $\mathscr{L}[\cos(bx)](\xi) = \int_0^{+\infty} e^{-x\xi}\cos(bx)dx = \frac{1}{2}\int_0^{+\infty} e^{(ib-\xi)x} + e^{(-ib-\xi)x}dx = \frac{1}{2}\left(\frac{1}{-ib+\xi} + \frac{1}{ib+\xi}\right) = \frac{ib+\xi+\xi-ib}{2(\xi^2+b^2)} = \frac{\xi}{\xi^2+b^2}$

$$\mathscr{L}[\cos(bx)](\xi) = \frac{\xi}{\xi^2+b^2} \quad (2)$$

RMF úkol č. 9

Lukáš Vácha

23. listopadu 2020

1 př č. 4

$\partial_t u + (4-x)\partial_x u = u; u(x,0) = e^{-x^2}$
Charakteristiky: $X'(t) = 4 - X(t)$
$X(t) = (4e^t + C)e^{-t} = 4 + Ce^{-t}$
$X_{x_0}(0) = x_0 = 4 + C$
$C = x_0 - 4$
$X_{x_0}(t) = 4 + (x_0 - 4)e^{-t} = 4 + x_0 e^{-t} - 4e^{-t}$
$x_0 = X_{x_0}(t)e^t - 4e^t + 4$
$x_0(x,t) = xe^t - 4e^t + 4$
Pro funkci v máme ODR: $v(t) = u(X(t), t)$
$v'(t) = \partial_x u \cdot X'(t) + \partial_t u = u(X(t), t) = v(t)$
$\frac{v'}{v} = 1$
$v(t) = Ce^t$
$v(0) = u(X(0), 0) = u(x_0, 0) = e^{-x_0^2}$
$v_{x_0}(t) = e^{-x_0^2 + t}$
$u(x,t) = v_{x_0}(t)\mid_{x_0 = x_0(x,t)} = e^{-(xe^t - 4e^t + 4)^2 + t}$
Řešení rovnice je tedy:

$$u(x,t) = e^{-(xe^t - 4e^t + 4)^2 + t} \tag{1}$$

RMF úkol č. 10

Lukáš Vácha

1. prosince 2020

1 př č. 1

$Lu(t) = u''(t) + 3u'(t) - 7u(t) = \sqrt{1-t^2}; u(0) = 1; u'(0) = -3$
Počítáme fundamentální systém s poč. pomínkami pomocí Laplaceovy transformace:, ozn. $\mathscr{L}[u(t)](p) = \tilde{u}(p)$
$L\mathscr{E} = \delta / \mathscr{L}$
$p^2\tilde{u}(p) - u'(0) - pu(0) - 3p\tilde{u}(p) - 3u(0) - 7\tilde{u}(p) = 1$
$(p^2 - 3p - 7)\tilde{u}(p) = p + 1$
$\tilde{u}(p) = \frac{p+1}{p^2-3p-7}$
$\mathscr{E}(t) = \mathscr{L}^{-1}[\frac{p+1}{p^2-3p-7}](t) = \mathscr{L}^{-1}[\frac{p-\frac{3}{2}}{(p-\frac{3}{2})^2-\frac{17}{2}} - \frac{5i}{\sqrt{34}}\frac{\sqrt{\frac{17}{2}}}{(p-\frac{3}{2})^2-\frac{17}{2}}](t) = \theta(t)e^{\frac{3}{2}t}[\cosh(\sqrt{\frac{17}{2}}t) + \frac{5}{\sqrt{34}}\sinh(\sqrt{\frac{17}{2}}t)]$
Klasické řešení:
$u(t) = \frac{1}{\theta(t)}(\mathscr{E} \star \theta\sqrt{1-\tau^2})(t) = \int_0^t \mathscr{E}(\tau)\sqrt{1-(t-\tau)^2}d\tau$

$$u(t) = \int_0^t e^{\frac{3}{2}\tau}\left[\cosh(\sqrt{\frac{17}{2}}\tau) + \frac{5}{\sqrt{34}}\sinh(\sqrt{\frac{17}{2}}\tau)\right]\sqrt{1-(t-\tau)^2}d\tau \qquad (1)$$

RMF úkol č. 10

Lukáš Vácha

9. prosince 2020

1 př č. 4

$\varphi(x) = \lambda \int_0^\pi \sin(x+y)\varphi(y)dy + \sin(x)$
$\varphi(x) = \lambda \int_0^\pi [\sin(x)\cos(y) + \cos(x)\sin(y)]\,\varphi(y)dy + \sin(x)$
$\varphi(x) = \lambda \sin(x) \int_0^\pi \cos(y)\varphi(y)dy + \cos(x) \int_0^\pi \sin(y)\varphi(y)dy + \sin(x)$
Při označní konstant:
$A = \lambda \int_0^\pi \cos(y)\varphi(y)dy + 1$
$B = \lambda \int_0^\pi \sin(y)dy$
Má rovnice tvar:
$\varphi(x) = A\sin(x) + B\cos(x)$
Vypočítáme konstanty A,B:
$A = \lambda \int_0^\pi \cos(y)[A\sin(y) + B\cos(y)]dy + 1$
$B = \lambda \int_0^\pi \sin(y)[A\sin(y) + B\cos(y)]dy$

$A = \lambda \int_0^\pi A\cos(y)\sin(y)dy + \int_0^\pi B\cos^2(y)dy + 1 = B\lambda\frac{\pi}{2} + 1;$[1]
$B = \lambda \int_0^\pi A\sin^2(y)dy + \int_0^\pi B\sin(y)\cos(y)dy = A\lambda\frac{\pi}{2}$

$\left(\begin{array}{cc|c} 1 & -\lambda\frac{\pi}{2} & 1 \\ \lambda\frac{\pi}{2} & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -\lambda\frac{\pi}{2} & 1 \\ 0 & (\lambda\frac{\pi}{2})^2 - 1 & -\lambda\frac{\pi}{2} \end{array}\right) \rightarrow \begin{cases} A = 1 + \frac{(\lambda\frac{\pi}{2})^2}{1-(\lambda\frac{\pi}{2})^2} = \frac{1}{1-(\lambda\frac{\pi}{2})^2} \\ B = \frac{\lambda\frac{\pi}{2}}{1-(\lambda\frac{\pi}{2})^2} \end{cases}$
Výsledek:

$$\varphi(x) = \frac{1}{1-(\lambda\frac{\pi}{2})^2}\sin(x) + \frac{\lambda\frac{\pi}{2}}{1-(\lambda\frac{\pi}{2})^2}\cos(x) \tag{1}$$

[1] $\int_0^\pi \sin(y)\cos(y)dy = \frac{1}{2}\int_0^\pi \sin(2y)dy = [\eta = 2y; 2d\eta = dy]\frac{1}{4}\int_0^{2\pi} \sin(\eta)d\eta = 0$
$\int_0^\pi \sin^2(y)dy = \int_0^\pi \cos^2(y)dy = \frac{\pi}{2}$

RMF úkol č. 12

Lukáš Vácha

13. prosince 2020

1 př č. 1

$\varphi(x,y) = \lambda \int_0^1 d\xi \int_0^3 d\eta x^2 \xi^2 y \eta \varphi(\xi,\eta) + xe^y$
Separace jádra:
$\varphi(x,y) = \lambda x^2 y \int_0^1 \int_0^3 \xi^2 \eta \varphi(\xi,\eta) d\eta d\xi + xe^y$
Označím konstantu:
$C = \int_0^1 \int_0^3 \xi^2 \eta \varphi(\xi,\eta) d\eta d\xi$
Pak má řešení tvar:
$\varphi(x,y) = C\lambda x^2 y + xe^y$
Určení konstanty C:
$C = \int_0^1 \int_0^3 \xi^2 \eta (C\lambda \xi^2 \eta + \xi e^\eta) d\eta d\xi = \int_0^1 (C\lambda \xi^4 \int_0^3 \eta^2 d\eta + \xi^3 \int_0^3 \eta e^\eta d\eta) d\xi = \int_0^1 (9C\lambda \xi^4 + \xi^3 (2e^3 + 1)) d\xi = \frac{9}{5} C\lambda + (2e^3+1)\frac{1}{4}$
$C = \frac{1}{1-\frac{9}{5}\lambda} (\frac{1}{2}\lambda e^3 + \frac{1}{4}) = \frac{10\lambda e^3 + 5}{20 - 36\lambda}$
s podmínkou: $\lambda \neq \frac{20}{36} = \frac{5}{8}$
Řešení je tedy:

$$\varphi(x,y) = \frac{10\lambda^2 e^3 + 5\lambda}{20 - 36\lambda} x^2 y + xe^y \tag{1}$$