RMF úkol č. 10

Lukáš Vácha

9. prosince 2020

1 př č. 4

$\varphi(x) = \lambda \int_0^\pi \sin(x+y)\varphi(y)dy + \sin(x)$
$\varphi(x) = \lambda \int_0^\pi [\sin(x)\cos(y) + \cos(x)\sin(y)]\,\varphi(y)dy + \sin(x)$
$\varphi(x) = \lambda \sin(x) \int_0^\pi \cos(y)\varphi(y)dy + \cos(x) \int_0^\pi \sin(y)\varphi(y)dy + \sin(x)$
Při označní konstant:
$A = \lambda \int_0^\pi \cos(y)\varphi(y)dy + 1$
$B = \lambda \int_0^\pi \sin(y)dy$
Má rovnice tvar:
$\varphi(x) = A\sin(x) + B\cos(x)$
Vypočítáme konstanty A,B:
$A = \lambda \int_0^\pi \cos(y)[A\sin(y) + B\cos(y)]dy + 1$
$B = \lambda \int_0^\pi \sin(y)[A\sin(y) + B\cos(y)]dy$

$A = \lambda \int_0^\pi A\cos(y)\sin(y)dy + \int_0^\pi B\cos^2(y)dy + 1 = B\lambda\frac{\pi}{2} + 1;$[1]
$B = \lambda \int_0^\pi A\sin^2(y)dy + \int_0^\pi B\sin(y)\cos(y)dy = A\lambda\frac{\pi}{2}$

$$\left(\begin{array}{cc|c} 1 & -\lambda\frac{\pi}{2} & 1 \\ \lambda\frac{\pi}{2} & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -\lambda\frac{\pi}{2} & 1 \\ 0 & (\lambda\frac{\pi}{2})^2 - 1 & -\lambda\frac{\pi}{2} \end{array}\right) \to \begin{cases} A = 1 + \frac{(\lambda\frac{\pi}{2})^2}{1-(\lambda\frac{\pi}{2})^2} = \frac{1}{1-(\lambda\frac{\pi}{2})^2} \\ B = \frac{\lambda\frac{\pi}{2}}{1-(\lambda\frac{\pi}{2})^2} \end{cases}$$

Výsledek:

$$\varphi(x) = \frac{1}{1-(\lambda\frac{\pi}{2})^2}\sin(x) + \frac{\lambda\frac{\pi}{2}}{1-(\lambda\frac{\pi}{2})^2}\cos(x) \qquad (1)$$

[1] $\int_0^\pi \sin(y)\cos(y)dy = \frac{1}{2}\int_0^\pi \sin(2y)dy = [\eta = 2y; 2d\eta = dy]\frac{1}{4}\int_0^{2\pi} \sin(\eta)d\eta = 0$
$\int_0^\pi \sin^2(y)dy = \int_0^\pi \cos^2(y)dy = \frac{\pi}{2}$