

RMF úkol do 9.10.

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1 příklad 2

$f \in \mathcal{D}'$?

- a. $(f, \varphi) = \varphi(\pi) = (\delta_\pi, \varphi)$
linearita: $(f, a\varphi + \psi) = (a\varphi + \psi)(\pi) = a\varphi(\pi) + \psi(\pi) = a(f, \varphi) + (f, \psi)$
spojitost: $\varphi_n \xrightarrow{\mathcal{D}} \varphi \Rightarrow \varphi_n \rightrightarrows \varphi \Rightarrow \lim_{n \rightarrow +\infty} \varphi_n = \varphi \Rightarrow \lim(f, \varphi_n) = \lim \varphi_n(\pi) = \varphi(\pi) = (f, \varphi)$
 $f \in \mathcal{D}'$
- b. $(f, \varphi) = (\varphi(0))^2$
linearita: $(f, a\varphi + \psi) = [(a\varphi + \psi)(0)]^2 = (a\varphi(0) + \psi(0))^2 = a^2\varphi^2(0) + 2a\varphi(0)\psi(0) + \psi^2(0) \neq a(f, \varphi) + (f, \psi)$
 $f \notin \mathcal{D}'$

2 příklad 3

$f \in \mathcal{D}'$?

- a. $(f, \varphi) = \varphi^{(n)}(\pi)$
linearita: $(f, a\varphi + \psi) = (a\varphi + \psi)^{(n)}(\pi) = a\varphi^{(n)}(\pi) + \psi^{(n)}(\pi) = a(f, \varphi) + (f, \psi)$
spojitost: $\varphi_n \xrightarrow{\mathcal{D}} \varphi \Rightarrow \forall \alpha D^\alpha \varphi_n \rightrightarrows D^\alpha \varphi \Rightarrow \lim_{k \rightarrow +\infty} \varphi_k^{(n)} = \varphi^{(n)} \Rightarrow \lim(f, \varphi_k) = \lim \varphi_k^{(n)}(\pi) = \varphi^{(n)}(\pi) = (f, \varphi)$
 $f \in \mathcal{D}'$
- b. $(f, \varphi) = \lim_{x \rightarrow +\infty} \varphi(x) = 0 = (0, \varphi)$
 $\mathcal{D}'$ je vektorový prostor $\Rightarrow 0 \in \mathcal{D}'$
 $f \in \mathcal{D}'$