

RMF úkol č. 6

Lukáš Vácha

8. listopadu 2020

1 př č. 2

Vypočítejte $(e^{-|t|} \star e^{-|t|})(x) = ?$:

$$|x - y| = \begin{cases} -(x - y); & x < y \\ (x - y); & x > y \end{cases}$$
$$x \cdot \operatorname{sgn}(x) = |x|$$

$$? = (e^{-|t|} \star e^{-|t|})(x) = \int_{-\infty}^{+\infty} e^{-|x-y|} e^{-|y|} dy =$$

$$\text{ad 1. } [x > 0] = \int_{-\infty}^0 e^{-(x-y)} e^y dy + \int_0^x e^{-(x-y)} e^{-y} dy + \int_x^{+\infty} e^{x-y} e^{-y} dy = e^{-x} \int_{-\infty}^0 e^{2y} dy + e^{-x} \int_0^x dy + e^x \int_x^{+\infty} e^{-2y} dy = \frac{1}{2} e^{-x} (1 - 0) + x e^{-x} - \frac{1}{2} e^x (0 - e^{-2x}) = (x + 1) e^{-x}$$

$$\text{ad 2. } [x < 0] = \int_{-\infty}^x e^{-(x-y)} e^y dy + \int_x^0 e^{x-y} e^y dy + \int_0^{+\infty} e^{x-y} e^{-y} dy = e^{-x} \int_{-\infty}^x e^{2y} dy + e^x \int_x^0 dy + e^x \int_0^{+\infty} e^{-2y} dy = \frac{1}{2} e^{-x} (e^{2x} - 0) - x e^x - \frac{1}{2} e^x (0 - 1) = (-x + 1) e^x$$

dohromady:

$$? = (x \cdot \operatorname{sgn}(x) + 1) e^{-x \cdot \operatorname{sgn}(x)} = (|x| + 1) e^{-|x|} \quad (1)$$