RMF úkol č. 9

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1 př č. 4

$\partial_t u + (4-x)\partial_x u = u; u(x,0) = e^{-x^2}$
Charakteristiky: $X'(t) = 4 - X(t)$
$X(t) = (4e^t + C)e^{-t} = 4 + Ce^{-t}$
$X_{x_0}(0) = x_0 = 4 + C$
$C = x_0 - 4$
$X_{x_0}(t) = 4 + (x_0 - 4)e^{-t} = 4 + x_0 e^{-t} - 4e^{-t}$
$x_0 = X_{x_0}(t)e^t - 4e^t + 4$
$x_0(x,t) = xe^t - 4e^t + 4$
Pro funkci v máme ODR: $v(t) = u(X(t),t)$
$v'(t) = \partial_x u \cdot X'(t) + \partial_t u = u(X(t),t) = v(t)$
$\frac{v'}{v} = 1$
$v(t) = Ce^t$
$v(0) = u(X(0),0) = u(x_0,0) = e^{-x_0^2}$
$v_{x_0}(t) = e^{-x_0^2+t}$
$u(x,t) = v_{x_0}(t) \mid_{x_0 = x_0(x,t)} = e^{-(xe^t - 4e^t + 4)^2 + t}$
Řešení rovnice je tedy:

$$u(x,t) = e^{-(xe^t - 4e^t + 4)^2 + t} \tag{1}$$