2.14 Rapidità

$v_2 = 0{,}85\,c$

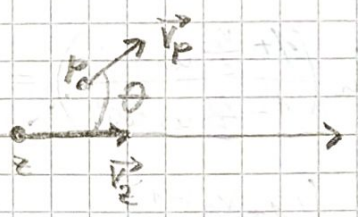
$\tilde{v}_2 = 0$

$\tilde{v}_p = 0{,}94\,c$

$v_p = ?$

$\theta_1 = 0°$

$\theta_2 = 180°$



$\beta = \frac{v}{c}$

$y = \frac{1}{2} \ln\left(\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right)$

- $\tilde{v}_2 = 0 \Rightarrow v_{CM} = v_2 = 0{,}85\,c$

- $\tilde{y}_p = y_p - y_{CM} \Rightarrow y_p = \tilde{y}_p + y_{CM}$

$\theta_1 = 0°$: $y_{CM} = \frac{1}{2} \ln\left(\frac{1+\frac{v_{CM}}{c}\cos 0°}{1-\frac{v_{CM}}{c}\cos 0°}\right) = \frac{1}{2} \ln\left(\frac{1+0{,}85}{1-0{,}85}\right) \doteq \underline{1{,}256}$

$\tilde{y}_p^{(1)} = \frac{1}{2} \ln\left(\frac{1+\frac{\tilde{v}_p}{c}\cos 0°}{1-\frac{\tilde{v}_p}{c}\cos 0°}\right) = \frac{1}{2} \ln\left(\frac{1+0{,}94}{1-0{,}94}\right) \doteq \underline{1{,}738}$

$\Rightarrow y_p^{(1)} \doteq \underline{2{,}994}$

$e^{2y} = \frac{1+\beta\cos\theta}{1-\beta\cos\theta}$

$e^{2y} - \beta\cos\theta\, e^{2y} = 1 + \beta\cos\theta$

$e^{2y} - 1 = \beta\cos\theta\,(e^{2y}+1)$

$\beta = \frac{1}{\cos\theta}\,\frac{e^{2y}-1}{e^{2y}+1}$

$v = \frac{c}{\cos\theta}\,\frac{e^{2y}-1}{e^{2y}+1}$

$\Rightarrow v_p^{(1)} = \frac{c}{\cos 0°}\,\frac{e^{2y_p^{(1)}}-1}{e^{2y_p^{(1)}}+1} = \underline{0{,}995\,c}$

$$v_p^{(1)} = \frac{\tilde{v}_p + v_{CM}}{1 + \frac{\tilde{v}_p v_{CM}}{c}} = \frac{0{,}94c + 0{,}85c}{1 + 0{,}94 \cdot 0{,}85} = \frac{1{,}79}{1{,}799}\, c \doteq \underline{0{,}995c}$$

$\theta_2 = 180°$: $\tilde{y}_p^{(2)} = \frac{1}{2} \ln\left(\frac{1 + \frac{v_p}{c}\cos 180°}{1 - \frac{v_p}{c}\cos 180°}\right) = \frac{1}{2}\ln\left(\frac{1 - \frac{v_p}{c}}{1 + \frac{v_p}{c}}\right) =$

$$= \frac{1}{2}\ln\left[\left(\frac{1 + \frac{v_p}{c}}{1 - \frac{v_p}{c}}\right)^{-1}\right] = -\frac{1}{2}\ln\left(\frac{1 + \frac{v_p}{c}}{1 - \frac{v_p}{c}}\right) = -\tilde{y}_p^{(1)} \doteq \underline{-1{,}738}$$

$$\Rightarrow y_p^{(2)} = -y_p^{(1)} + y_{CM} \doteq \underline{-0{,}482}$$

$$\Rightarrow v_p^{(2)} = \frac{c}{\cos 180°} \frac{e^{2y_p^{(2)}} - 1}{e^{2y_p^{(2)}} + 1} \doteq \underline{0{,}448\,c}$$

$$v_p^{(2)} = \frac{\tilde{v}_p - v_{CM}}{1 - \frac{\tilde{v}_p v_{CM}}{c}} = \frac{0{,}94c - 0{,}85c}{1 - 0{,}94 \cdot 0{,}85} = \frac{0{,}09c}{0{,}201} \doteq \underline{0{,}448c}$$