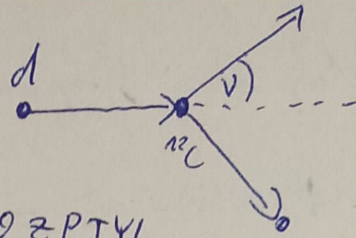
2. kapitola, příklad 2)

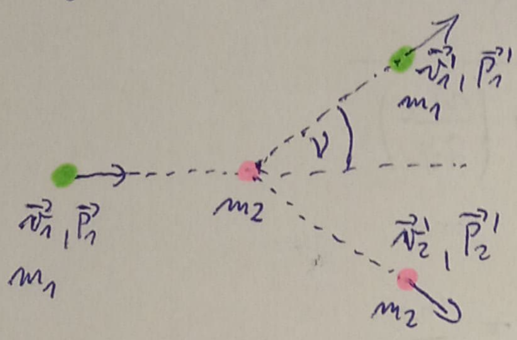
$m(d) = 2{,}013$

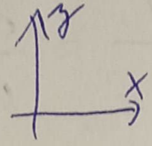
$m(^{12}C) = 11{,}997$



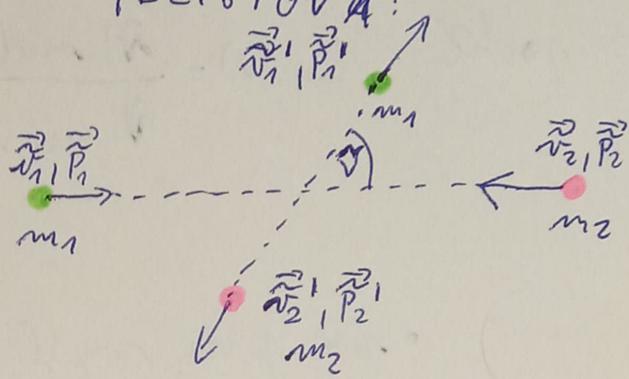
PRUŽNÝ ROZPTYL

→ LABORATORNÍ:





TĚŽIŠŤOVÁ:



- tak aby těleso s m_2 bylo v klidu $\rightarrow \vec{v}_2 = 0$

- tak aby celková hybnost byla nulová $\rightarrow \vec{\vec{P}}_{CELK} = \vec{0}$

→ rychlost těžiště v laboratorní soustavě (ve směru nalétající částice):

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{m_1\vec{v}_1}{m_1+m_2}$$

→ v těžišťové soustavě se částice pohybují proti sobě:

$$\vec{\vec{v}}_1 = \vec{v}_1 - \vec{v}_{CM} = \frac{m_2\vec{v}_1}{m_1+m_2} \quad ; \quad \vec{\vec{v}}_2 = \vec{v}_2 - \vec{v}_{CM} = -\frac{m_1\vec{v}_1}{m_1+m_2}$$

↳ tedy $\vec{\vec{P}}_1 = m_1\vec{\vec{v}}_1 = \frac{m_1 m_2}{m_1+m_2}\vec{v}_1 \quad ; \quad \vec{\vec{P}}_2 = m_2\vec{\vec{v}}_2 = -\frac{m_1 m_2}{m_1+m_2}\vec{v}_1$

$$\rightarrow \vec{\vec{P}}_{CELK} = \vec{\vec{P}}_1 + \vec{\vec{P}}_2 = \vec{0}$$

→ vztah mezi komponentami rychlostí:

$$\begin{cases} x: \ v_1' \cos(\vartheta) - v_{CM} = \tilde{v}_1' \cos(\tilde{\vartheta}) \longrightarrow v_1' \cos(\vartheta) = \tilde{v}_1' \cos(\tilde{\vartheta}) + v_{CM} \\ y: \ v_1' \sin(\vartheta) = \tilde{v}_1' \sin(\tilde{\vartheta}) \end{cases}$$

→ podíl: $$\tan(\vartheta) = \frac{\tilde{v}_1' \sin(\tilde{\vartheta})}{\tilde{v}_1' \cos(\tilde{\vartheta}) + v_{CM}} = \frac{\sin(\tilde{\vartheta})}{\cos(\tilde{\vartheta}) + \left(\frac{v_{CM}}{\tilde{v}_1'}\right)}$$

• pro pružný rozptyl platí $\xi = \frac{v_{CM}}{\tilde{v}_1'} = \frac{m_1}{m_2}$

$$\Longrightarrow \boxed{\tan(\vartheta) = \frac{\sin(\tilde{\vartheta})}{\cos(\tilde{\vartheta}) + \left(\frac{m_1}{m_2}\right)}} \quad *$$

a) $$\operatorname{arctg}\left(\frac{\sin(\tilde{\vartheta})}{\cos(\tilde{\vartheta}) + \left(\frac{m_1}{m_2}\right)}\right) = \vartheta = \operatorname{arctg}\left(\frac{\sin(45^\circ)}{\cos(45^\circ) + \left(\frac{2{,}013}{11{,}997}\right)}\right) = \underline{\underline{38{,}9^\circ}}$$

* možno přepsat jako: $$\cos(\vartheta) = \frac{\cos(\tilde{\vartheta}) + \xi}{\sqrt{1 + 2\xi\cos(\tilde{\vartheta}) + \xi^2}}$$

→ odvození minulého vztahu:

$$\cos^2(\vartheta)\left(1 + 2\xi\cos(\tilde{\vartheta}) + \xi^2\right) = \cos^2(\tilde{\vartheta}) + 2\xi\cos(\tilde{\vartheta}) + \xi^2$$

$$\cos^2(\tilde{\vartheta}) + \cos(\tilde{\vartheta})\left(2\xi\sin^2(\vartheta)\right) + \left(\xi^2 - \cos^2(\vartheta) - \cos^2(\vartheta)\xi^2\right) = 0$$

$$\rightarrow \cos(\tilde{\vartheta}) = \frac{1}{2}\left(-2\xi\sin^2(\vartheta) \pm \sqrt{4\xi^2\sin^4(\vartheta) - 4\xi^2 + 4\cos^2(\vartheta) + 4\cos^2(\vartheta)\xi^2}\right)$$

$$= -\xi\sin^2(\vartheta) \pm \sqrt{\xi^2\left(1 - \cos^2(\vartheta)\right)^2 - \xi^2 + \cos^2(\vartheta) + \cos^2(\vartheta)\xi^2} \Longrightarrow$$

$$= \sqrt{\xi^2 - 2\xi^2\cos^2(\nu) + \xi^2\cos^4(\nu) - \xi^2 + \cos^2(\nu) + \cos^2(\nu)\xi^2} =$$

$$= \sqrt{\cos^2(\nu) + \xi^2\cos^4(\nu) - \xi^2\cos^2(\nu)} = \cos(\nu)\sqrt{1 + \xi^2\cos^2(\nu) - \xi^2} =$$

$$= \cos(\nu)\sqrt{1 - \xi^2\sin^2(\nu)} \Rightarrow$$

$$\boxed{\cos(\nu) = -\xi\sin^2(\nu) + \cos(\nu)\sqrt{1 - \xi^2\sin^2(\nu)}}$$

$\Rightarrow$ b) $\underline{\underline{\nu = 89{,}5^\circ}}$

c) $\underline{\underline{\nu = 99{,}7^\circ}}$