Slovníček množin a zobrazení

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1 Množiny, prostory

1. Podmnožina: $A \subset B \Leftrightarrow (x \in A \Rightarrow x \in B)$
2. Prázdná množina: $\varnothing = \{\}; (\forall x)(x \notin \varnothing)$
3. Potenční množina: $\mathcal{P}(X) = \{A \mid A \subset X\}$
4. Průnik: $A \cap B = \{x \mid x \in A \wedge x \in B\}$
5. Sjednocení: $A \cup B = \{x \mid x \in A \vee x \in B\}$
6. Rozdíl: $X \backslash A = \{x \in X \mid x \notin A\}$
7. Kartézský součin: $A \times B = \{(x, y) \mid x \in A, y \in B\}$
8. Mocnina: $A^n = \bigtimes^n A$
9. Funkce: $f : A \to B = \{(x, y) \in A \times B \mid (x_1, y) \in f \wedge (x_2, y) \in f \Rightarrow x_1 = x_2\}$
10. Definiční obor: $Dom(f) = \{x \in A \mid \exists f(x)\}$
11. Obor hodnot: $Ran(f) = \{y \in B \mid (\exists x \in A)(y = f(x))\} = Im(f)$
12. Přirozená čísla: $\mathbb{N} = \{1, 2, 3, ...\} = \mathbb{Z}_+$
13. $\hat{n} = \{i \mid i \in \mathbb{N}, i \leqslant n\} = \{1, 2, 3, ..., n\}$
14. $\mathbb{N}_0 = \mathbb{N} \cup \{0\} = \mathbb{Z}_0^+ = (\mathbb{N}, \sigma, 0) = \{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}, \{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}, ...\}$
15. Celá čísla: $\mathbb{Z} = \{z \mid |z| \in \mathbb{N}_0\}$
16. Racionální čísla: $\mathbb{Q} = \{\frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{N}, p \perp q\}$
17. Multiindexy: $\mathbb{N}^n = \mathbb{Z}_+^n = \{(a_1, a_2, ..., a_n) \mid a_i \in \mathbb{N}, \forall i \in \hat{n}\}$
18. Reálná čísla: $\mathbb{R} = \{x \mid \exists (x_n)_{n=1}^{+\infty}, \lim_{n \to +\infty} x_n = x, x_n \text{ Cauchy}\} \sim \overline{\mathbb{Q}}$
    $\mathbb{R}^* = \mathbb{R} \cup \{\pm\infty\}$
19. Komplexní čísla: $\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R}, i = \sqrt{-1}\}$

20. Prvočísla: $\mathbb{P} = \{p \in \mathbb{N} \mid (\forall n \in \{2, 3, ..., p-1\})(p \perp n)\}$
21. Topologický prostor: $(X, \tau), \tau \subset \mathcal{P}(X), \varnothing, X \in \tau, \bigcap_{\text{spočet.}} A_i \in \tau, \bigcup A_\alpha \in \tau$
22. Metrický prostor: $(X, \rho), \rho : X \times X \to \mathbb{R}_0^+$ metrika
23. Koule: $B(x, R) = \{y \in X \mid \rho(x, y) < R\}$
24. Kompaktní prostor: Každé otevřené pokrytí má konečené podpokrytí.
25. Úplný prostor: Každá Cauchyovská posloupnost konverguje v prostoru.
26. Souvislý prostor: Každá spojitá dráha leží celá v prostoru.
27. Separabilní prostor: $(\exists A \subset X)(\overline{A} = X \wedge |A| < +\infty)$
28. Těleso: $T = \mathbb{C}, \mathbb{R}, \mathbb{Q}$
29. Vektrorový prostor: $(V, T, \oplus, \odot)$, axiomy...
30. Lineární obal: $span(x_1, ..., x_n) = [x_1, ..., x_n]_\lambda = \{x \in V \mid (\exists \alpha^n \in T^n)$
$(x = \sum_{i=1}^n \alpha_i x_i)\}$
31. Pologrupa: $(G, *), (a * b) * c = a * (b * c)$
32. Grupoid: $(G, *), * : G \times G \to G$
33. Monoid: $(G, *, e), (a * b) * c = a * (b * c) \wedge a * e = e * a = a$
34. Nuloid: $(G, *, z), (a * b) * c = a * (b * c) \wedge a * z = z * a = z$
35. Grupa: $(G, *, e, ^{-1}), (a * b) * c = a * (b * c) \wedge a * e = e * a = a \wedge a^{-1} * a = a * a^{-1} = e$
36. Kvazigrupa: $(G, ^{-1}), a * x = b \wedge y * a = b$
37. Lupa: $(G, ^{-1}, e), a * e = e * a = a$
38. Pre-Hilbertův prostor: $(V, \langle . \mid . \rangle)$
39. Normovaný prostor: $(V, \|.\|)$
40. Hilbertův prostor: $\mathcal{H} = \mathfrak{H} = \mathscr{H} = (V, \langle . \mid . \rangle)$ úplný
41. Banachův prostor: $\mathcal{B}$ Hilbertův a separabilní
42. $\mathbb{R}^n = (\{(x_1, x_2, ..., x_n) \mid x_i \in \mathbb{R}, \forall i \in \hat{n}\}, \mathbb{R}, \oplus, \odot)$
43. $C^k(\mathbb{R}^n) = \{f : \mathbb{R}^n \to \mathbb{R} \mid \exists \partial_{x_i}^j f$ spojité, $\forall i \in \hat{n}, \forall j \in \hat{k}\}$
$C^0 = \{f$ spojité$\}, C^\infty = \{f \mid \forall k \in \mathbb{N}, \exists \partial^k f\}, C^\omega = \{f \mid f(x) = \sum_{k=0}^{+\infty} \frac{f^{(k)}(0)}{k!} x\}$
44. Nosič: $suppf = \overline{\{x \in Dom(f) \mid f(x) \neq 0\}}$
45. Testovací funkce: $\mathscr{D}(\mathbb{R}^n) = \{\varphi : \mathbb{R}^n \to \mathbb{C} \mid suppf$ omezený $\wedge \varphi \in C^\infty(\mathbb{R}^n)\}$

46. Zobecněné funkce: $\mathscr{D}' = \{f : \mathscr{D} \to \mathbb{C} \mid (f, \alpha\varphi + \psi) = \alpha(f, \varphi) + (f, \psi) \wedge \lim(f, \varphi_n) = (f, \lim \varphi_n)\}$

47. Schwarzův prostor: $\mathscr{S} = \{\varphi : \mathbb{R}^n \to \mathbb{C} \mid (\forall \alpha, \beta \in \mathbb{Z}^n_+)(sup_{x \in \mathbb{R}^n} \left|x^\alpha D^\beta f(x)\right| < +\infty) \wedge (\varphi \in C^\infty(\mathbb{R}^n))\}$

48. Temperované distribuce: $\mathscr{S}' = \{f : \mathscr{S} \to \mathbb{C} \mid (f, \alpha\varphi + \psi) = \alpha(f, \varphi) + (f, \psi) \wedge \lim(f, \varphi_n) = (f, \lim \varphi_n)\}$

49. $\mathcal{L}^p(\mathbb{R}^n) = \{f : \mathbb{R}^n \to \mathbb{R} \mid \int_{\mathbb{R}^n} |f|^p < +\infty\}$

50. $L^p(\mathbb{R}^n) = \mathcal{L}/_\sim; f \sim g \Leftrightarrow f(x) = g(x), \forall x \in (\mathbb{R}^n \backslash M), \mu(M) = 0$
$\langle f \mid g \rangle = \int_{\mathbb{R}}^n \overline{f}(x) g(x) dx$

51. $L^p_{loc.}(\mathbb{R}^n) = \{f : \mathbb{R}^n \to \mathbb{R} \mid (\forall x \in \mathbb{R}^n)(\exists H_x)(f \in L^p(H_x)\}$

52. Jádro: $\ker(f) = \{x \in Dom(f) \mid f(x) = 0 = e$ (Grupa)$\}$

53. Soubor základních funkcí: $\mathcal{H}(\mathcal{X}) = \{f : X \to \mathbb{R} \mid$ omezené, tvoří VP; $h \in \mathcal{H} \Rightarrow |h| \in \mathcal{H}\}$

54. $\Lambda^+(X) = \{f : X \to \mathbb{R}^* \mid (\exists (h_n)_{n=1}^{+\infty} \subset \mathcal{H}(X))(h_n < h_{n+1} \wedge \lim h_n = f\ s.v.)\}$

55. $\Lambda(X) = \{\varphi \mid (\exists f, g \in \Lambda^+(X))(f \in \mathcal{L}^+(X) \vee g \in \mathcal{L}^+(X))(\varphi = f - g\ s.v.)\}$

56. $\mathcal{L}^+(X) = \{f \in \Lambda^+(X) \mid (\exists c > 0)(\forall n \in \mathbb{N})(I h_n \leqslant c)\}$

57. $\mathcal{L}(X) = \{\varphi \mid (\exists f, g \in \mathcal{L}^+(X))(\varphi = f - g\ s.v.)\}$

58. Obraz: $Im_f(M) = \{f(x) \in Ran(f) \mid x \in M \subset Dom(f)\}$

59. Resolventa: $\rho(A) = \{\lambda \in T \mid \det(A - \lambda I) \neq 0\} =$
$= \{\lambda \in T \mid \det(A - \lambda I)$ je bijekce$\}$

60. Spektrum: $\sigma(A) = \{\lambda \in T \mid \det(A - \lambda I) = 0\} =$
$= \{\lambda \in T \mid \det(A - \lambda I)$ není bijekce$\} = T \backslash \rho(A)$

61. Lineární zobrazení: $\mathcal{L}(X, Y) = \{f : X \to Y \mid f(\alpha x + y) = \alpha f(x) + f(y)\}$

62. $GL(n, T) = \{\mathbb{A} \in T^{n,n} \mid \det \mathbb{A} \neq 0\}$

63. $SL(n, T) = \{\mathbb{A} \in T^{n,n} \mid \det \mathbb{A} = 1\}$

64. $O(n, T) = \{\mathbb{A} \in T^{n,n} \mid \mathbb{A}^T = \mathbb{A}^{-1}\}$

65. Liova algebra: $\mathcal{A} = (V, [.,.]); [.,.] : V \times V \to V$ bilineární a $[[F_1 + F_2], G] + [[F_2, G], F_1] + [[G, F_1], F_2] = 0$

66. Afinní prostor: $(E, \overrightarrow{E}, -); .-. : E \times E \to \overrightarrow{E}$ bijekce; $(x - y) \oplus (y - z) = (x - z)$

67. Vnější mocnina: $\Lambda^k(V^n) = \{\omega \mid \omega$ k-lineární anisymetrická forma$\}$

68. OG doplněk: $M^{\perp} = \{\overrightarrow{x} \in V \mid (\forall y \in M)(\langle x \mid y \rangle = 0)\} \subset\subset V$
69. ON báze: $\mathscr{B}(V^n) = (e_i)_{i=1}^n; \langle e_i \mid e_j \rangle = \delta_{ij}$
70. Varieta: $W = \{x \in \mathbb{R}^n \mid \phi : H_x \to \mathbb{R}^m \in C^q \wedge M \cap H_x = \{x \in H_x \mid \phi(x) = 0\} \wedge Rank(\phi'(x)) = m\}$
71. σ-algebra: $\Sigma \subset \mathcal{P}(X); X \in \Sigma \wedge A \in \Sigma \Rightarrow (X \backslash A) \in \Sigma \wedge \bigcup_{\text{spočet.}} A_i \in \Sigma$
72. $\mathscr{D}'_{reg.} = \{\tilde{f} \in \mathscr{D}' \mid (\exists f \in L^1_{loc.}(\mathbb{R}^n))(\forall \varphi \in \mathscr{D}(\mathbb{R}^n))((f, \varphi) = \int_{\mathbb{R}^n} f(x)\varphi(x)dx)\}$
73. $\mathscr{S}'_{reg.} = \{\tilde{f} \in \mathscr{S}' \mid (\exists f \in L^1(\mathbb{R}^n))(\forall \varphi \in \mathscr{S}(\mathbb{R}^n))((f, \varphi) = \int_{\mathbb{R}^n} f(x)\varphi(x)dx)\}$
74. Fázový prostor: $\Gamma = \{(q, p) \mid q \in \mathbb{R}^n; p \in \mathbb{R}^n\}$ je affiní
75. Konfigurační prostor: $C = \Gamma_q = \{q \mid q \in \mathbb{R}^n\}$ je diferenciální varieta
76. Minkovského časoprostor: $(\mathbb{R}^2, \langle x \mid y \rangle = x^* \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} y) \ni \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$
77. Tenzorový součin: $A \otimes B = \{x \otimes y \mid x \in A; y \in B\}$
78. Vnitřek mžy: $M^{\circ} = \bigcup_{B \subset M B \in \tau} B$
79. Hranice mžy: $\partial M = \dot{M} = \{x \in X \mid x \in X \backslash (M^{\circ} \cup (X \backslash M)^{\circ})\}$
80. Modulo n: $mod\ n = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x \equiv y mod\ n; x = k_1 n + r \wedge y = k_2 n + r; k_1, k_2 \in \mathbb{Z}\}$

2 Funkce, zobrazení

1. Identita: $id : A \to A; id(x) = x$
2. Kardinalita: card: (množina všech množin)$\to (\mathbb{N}_0 \cup \aleph_{\mathbb{N}_0}); card(A) = |A| =$ počet prvků A
   $|\varnothing| = 0, |\hat{n}| = n, |\mathbb{N}| = \aleph_0, |\mathbb{R}| = \aleph_1, ...$
3. Skládání: $\circ : \{f : B \to C\} \times \{f : A \to B\} \to \{f : A \to C\}; (f \circ g)(x) = f(g(x))$
4. Iterace: $f^n : \times^n \{f : A \to A\} \to \{f : A \to A\}; f^n = \bigcirc^n f = f \circ f \circ ... \circ f$
5. Inverze: $f^{-1} : \{f : A \to B \mid \text{ prostá}\} \to \{f : B \to A \mid \text{ prostá}\}; f^{-1} \circ f = f \circ f^{-1} = id$
6. Maximum: $\max : \{f\} \to Ran(f); max f = f(x_0), (\forall x \neq x_0)(f(x) < f(x_0))$
7. Minimum: $\min : \{f\} \to Ran(f); min f = f(x_0), (\forall x \neq x_0)(f(x) > f(x_0))$

8. Supremum: sup : $\{f\} \to \overline{Ran(f)}; supf = y, (\forall x \in Dom(f))(f(x) < x) \wedge (\forall y' < y)(\exists x \in Dom(f))(f(x)) > y')$

9. Infimum: inf : $\{f\} \to \overline{Ran(f)}; inff = y, (\forall x \in Dom(f))(f(x) > x) \wedge (\forall y' > y)(\exists x \in Dom(f))(f(x)) < y')$

10. Posloupnost: $a_n : \mathbb{N} \to \mathbb{C}$

11. Funkční posloupnost: $f_n : \mathbb{N} \to \{f\}$

12. $\overline{.} : \mathbb{C} \to \mathbb{C}; \overline{z} = a - bi, z = a + bi, a, b \in \mathbb{R}$

13. $\Re : \mathbb{C} \to \mathbb{R}; \Re z = \frac{z+\overline{z}}{2}$

14. $\Im : \mathbb{C} \to \mathbb{R}; \Im z = \frac{z-\overline{z}}{2i}$

15. $konst. : \mathbb{R} \to \{konst.\}; konst.(x) = konst.$

16. $|.| : \mathbb{R} \to \mathbb{R}_0^+; |x| = \begin{cases} x; x \geqslant 0 \\ -x; x < 0 \end{cases}$

17. $\Theta : \mathbb{R} \to \{0, 1\}; \Theta(x) = \begin{cases} 1; x > 0 \\ 0; x < 0 \end{cases}$

18. $sgn : \mathbb{R} \to \{-1, 0, 1\}; sgn(x) = \begin{cases} 1; x > 0 \\ 0; x = 0 \\ -1; x < 0 \end{cases}$

19. $\chi_A : \mathbb{R} \to \{0, 1\}; \chi_A(x) = \begin{cases} 1; x \in A \\ 0; x \notin A \end{cases}$
    $\chi_{(-a,a)}(x) = \Theta(a - |x|)$

20. Polynomy: $f : \mathbb{R}^n \to \mathbb{R}; f(x) = \sum_{i=1}^{n} \sum_{j=0}^{N} c_{ij} x_i^j$

21. Odmocnina: $\sqrt{.} : \mathbb{R}_+ \to \mathbb{R}_+; \sqrt{x} = (x^2)^{-1} = x^{\frac{1}{2}}$

22. Sinus: $\sin : \mathbb{R} \to (-1; 1); \sin(x) = \sum_{n=0}^{+\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$
    $\arcsin : [-1; 1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]; \arcsin(x) = (\sin(x))^{-1}$

23. Cosinus: $\cos : \mathbb{R} \to (-1; 1); \cos(x) = \sum_{n=0}^{+\infty} \frac{(-1)^{n+1} x^{2n}}{(2n)!}$
    $\arccos : [-1; 1] \to [0, \pi]; \arccos(x) = (\cos(x))^{-1}$

24. Tangens: $tg : \mathbb{R} \backslash \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\} \to \mathbb{R}; tg(x) = \frac{\sin(x)}{\cos(x)}$
    $arctg : \mathbb{R} \to (-\frac{\pi}{2}, \frac{\pi}{2}); arctg(x) = (tg(x))^{-1}$

25. Cotangens: $cotg : \mathbb{R} \backslash \{k\pi \mid k \in \mathbb{Z}\} \to \mathbb{R}; cotg(x) = \frac{\cos(x)}{\sin(x)}$
    $arccotg : \mathbb{R} \to (0, \pi); arccotg(x) = (cotg(x))^{-1}$

26. Exponenciála:

(a) $\exp : \mathbb{R} \to \mathbb{R}_+; \exp x = e^x = \sum_{n=0}^{+\infty} \frac{x^n}{n!}$

(b) $\exp : \mathbb{C} \to \mathbb{C}; \exp z = e^z = e^{\Re z}(\cos \Im z + i \sin \Im z)$

27. Logaritmus: $\ln : \mathbb{R}_+ \to \mathbb{R}; \ln x = (e^x)^{-1}$

28. $\sinh : \mathbb{R} \to \mathbb{R}; \sinh(x) = \frac{e^x - e^{-x}}{2}$
$argsinh : \mathbb{R} \to \mathbb{R}; argsinh(x) = \ln(x + \sqrt{x^2 + 1}$

29. $\cosh : \mathbb{R} \to [1; +\infty); \cosh(x) = \frac{e^x + e^{-x}}{2}$
$argsinh : [1; +\infty) \to \mathbb{R}; argcosh(x) = \ln(x + \sqrt{x^2 - 1}$

30. $tgh : \mathbb{R} \to (-1; 1); tgh(x) = tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
$argtgh : (-1; 1) \to \mathbb{R}; argtgh(x) = \frac{1}{2} \ln(\frac{1+x}{1-x})$

31. $cotgh : \mathbb{R} \to \mathbb{R} \backslash (-1; 1); cotgh(x) = coth(x) = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$
$argcotgh : \mathbb{R} \backslash (-1; 1) \to \mathbb{R}; argcotgh(x) = \frac{1}{2} \ln(\frac{x+1}{x-1})$

32. Gamma: $\Gamma : \mathbb{R}_+ \to \mathbb{R}; \Gamma(x) = \int\limits_0^{+\infty} t^{x-1} e^{-t} dt$

33. Beta: $\mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}; B(x, y) = \int\limits_0^1 t^{x-1} (1-t)^{y-1} dt$

34. Celá část: $\lfloor . \rfloor : \mathbb{R} \to \mathbb{Z}; \lfloor x \rfloor = n, n < x < n + 1$

35. $\lceil . \rceil : \mathbb{R} \to \mathbb{Z}; \lceil x \rceil = n, n - 1 < x < n$

36. $nsd : \mathbb{N}^N \to \mathbb{N}; nsd(n_1, n_2, ..., n_N) = max\{n \in \mathbb{N} \mid n/n_i, \forall i \in \hat{N}\}$

37. $nsn : \mathbb{N}^N \to \mathbb{N}; nsn(n_1, n_2, ..., n_N) = min\{n \in \mathbb{N} \mid n_i/n, \forall i \in \hat{N}\}$

38. $! : \mathbb{N} \to \mathbb{N}; n! = \prod_{i=1}^n i = 1 \cdot 2 \cdot ... \cdot n$

39. Limita:

(a) $\lim(X, \rho) : \{(x_n)_{n=1}^{+\infty} \mid x_n \in (X, \rho), \forall n \in \mathbb{N}\} \to \overline{X}; \lim x_n = x,$
$(\forall \varepsilon > 0)(\exists n_0 \in \mathbb{N})(\forall n \in \mathbb{N}, n > n_0)(\rho(x_n, x) < \varepsilon)$

(b) $\lim(\mathbb{R}^n) : \{f : \mathbb{R}^n \to \mathbb{R}\} \to \overline{R}^n; \lim_{x \to x_0} f(x) = y,$
$(\forall \varepsilon > 0)(\exists \delta > 0)(\forall x, \|x - x_0\|)(|f(x) - y| < \varepsilon)$

(c) $\lim(\mathscr{D}) : (\varphi_n)_{n=1}^{+\infty} \subset \mathscr{D} \to \mathscr{D}; \lim \varphi_n = \varphi,$
$(\forall \alpha \in \mathbb{Z}_+^n)(D^\alpha \varphi_n \overset{\mathbb{R}^n}{\rightrightarrows} D^\alpha \varphi) \wedge (\exists R > 0)(\forall n \in \mathbb{N})(supp\ \varphi_n \subset B(0, R))$

40. $\overset{\mathbb{R}^n}{\rightrightarrows}: \{f : \mathbb{R} \times \mathbb{N} \to \mathbb{R}\} \to \{f : \mathbb{R} \to \mathbb{R}\}; f_n \overset{\mathbb{R}^n}{\rightrightarrows} f \Leftrightarrow$
$\lim_{n \to +\infty} \sup_{x \in \mathbb{R}^n} |f_n(x) - f(x)| = 0$

41. metrika: $\rho : X \times X \to \mathbb{R}_0^+; \rho(x, y) = \rho(y, x), \rho(x, y) = 0 \Leftrightarrow x = y,$
$\rho(x, y) \leqslant \rho(x, z) + \rho(z, y)$

42. norma: $\|.\| : V \to \mathbb{R}_0^+; \|x\| = 0 \Leftrightarrow x = \overrightarrow{0}, \|\alpha x\| = |\alpha| \, \|x\|$,
$\|x+y\| = \|x\| + \|y\|$

43. $\oplus : V \times V \to V$; axiomy

44. $\odot : T \times V \to V$, axiomy

45. Skalární součin: $\langle . \mid . \rangle : V \times V \to T$; lineární a antilineární, antisymetrický, pozitivně definitní

46. Parciální derivace: $\partial_{x_i} : \{f : \mathbb{R}^n \to \mathbb{R}^m\} \to \{f : \mathbb{R}^n \to \mathbb{R}^m\}; \partial_{x_i} f(x) = \frac{\partial f(x)}{\partial x_i} = \lim \frac{(f(x_i + \frac{1}{n}) - f(x_i)}{\frac{1}{n}}$

47. k-tá parciální derivace: $\partial_{x_i}^k : \{f : \mathbb{R}^n \to \mathbb{R}^m\} \to \{f : \mathbb{R}^n \to \mathbb{R}^m\}; \partial_{x_i}^k f = \frac{\partial^k f}{\partial x_i^k} = (\partial_{x_i})^k$

48. Derivace:

$$(.)'\{f : \mathbb{R}^n \to \mathbb{R}^m\} \to \mathscr{L}(\mathbb{R}^n, \mathbb{R}^m); \lim_{x \to x_0} \frac{f(x) - f(x_0) - f'(x - x_0)}{\|x - x_0\|} = 0$$

$f'(x) = \nabla \otimes f$[1]

49. Integrál: $\int : \{f \in C^0(\mathbb{R})\} \to \{f \in C^1(\mathbb{R})\}; (\int f(x)dx)' = f(x)$

50. $I : \Lambda^+ \to \mathbb{R} \cup \{\pm\infty\}; If = \lim Ih_n$

51. $\int_M : \Lambda \to \mathbb{R}; \int_M f = Ig_1 - Ig_2 = \lim_{n \to +\infty} \sum_{i=0}^n \frac{1}{n} f(\xi_i), \xi_i \in (a + \frac{b-a}{n} i; a + \frac{b-a}{n}(i+1), M = (a, b)$

52. $D^\alpha : \{f : \mathbb{R}^n \to \mathbb{R}^m\} \to \{f : \mathbb{R}^n \to \mathbb{R}^m\}; D^\alpha = \partial_{x_1}^{\alpha_1} \partial_{x_2}^{\alpha_2} ... \partial_{x_n}^{\alpha_n}$

53. Delta: $\delta_{x_0} : \mathscr{D} \to \mathbb{C}; (\delta_{x_0}, \varphi) = \varphi(x_0)$

54. $\mathcal{P}\frac{1}{x} : \mathscr{D} \to \mathbb{C}; (P\frac{1}{x}, \varphi) = \lim_{\varepsilon \to 0^+} \int_{\mathbb{R} \setminus (-\varepsilon; \varepsilon)} \frac{1}{x} \varphi(x) dx = \lim_{\varepsilon \to 0^+} \int_\varepsilon^{+\infty} \frac{\varphi(x) - \varphi(-x)}{x}$

55. Tentorový součin: $\otimes : \{f : \mathbb{R}^n \to \mathbb{R}\} \times \{f : \mathbb{R}^m \to \mathbb{R}\} \to \{f : \mathbb{R}^{n+m} \to \mathbb{R}\}; f(x) \otimes g(y) = f(x)g(y) = (f, (g, \varphi(x, y)))$

56. Konvoluce: $\star : L^1(\mathbb{R}^n) \times L^1(\mathbb{R}^n) \to L^1(\mathbb{R}^n); (f \star g)(x) = \int_{\mathbb{R}^n} f(x-y) g(y) dy = (f(x) \otimes g(x), \varphi(x+y))$

57. Fourierova transformace: $\mathscr{F} : \mathscr{S} \to \mathscr{S}; \mathscr{F}[f(x)](\xi) = \int_{\mathbb{R}^n} e^{ix\xi} f(x) dx = (f(\xi), \mathscr{F}[\varphi(x)](\xi))$

58. Sochockého distribuce: $\frac{1}{x+0i} : \mathscr{D} \to \mathbb{C}; (\frac{1}{x+0i}, \varphi) = \lim_{\varepsilon \to 0^+} \int_{\mathbb{R}} \frac{1}{x + \varepsilon i} \varphi(x) dx =$

[1]Nabla: $\nabla = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix}$

59. Homomorfismus: $f : \mathcal{A} \to \mathcal{B}; (\forall g \in \mathcal{A})(f(g(x)) = g(f(x)))$

60. Monomorfismus: prostý homomorfismus, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

61. Epimorfismus: surjektvivní homomorfismus (na)

62. Izomorfismus: homomorfismus bijekce

63. Endomorfismus: homomorfismis, $\mathcal{A} = \mathcal{B}$

64. Automorfismus: endomorfismus bijekce

65. Permutace: $\pi : \hat{n} \to \hat{n}; \pi$ bijekce

66. Determinant: $\det : T^{n,n} \to T; \det(A) = \sum_{\pi \in S_n} \prod_{i=1}^{n} A_{i\pi(i)} = \sum_{\pi \in S_n} A_{1\pi(1)} A_{2\pi(2)} A_{3\pi(3)} ... A_{n\pi(n)}$

67. Dimenze: $\dim : V \to \mathbb{N}_0 \cup \{+\infty\}; \dim V = \max\{|M| \mid x_i \in M \;\; LN\} = |\mathscr{B}|, \mathscr{B}$ báze V

68. Násobení matic: $\cdot : \mathbb{R}^{n,m} \times \mathbb{R}^{m,k} \to \mathbb{R}^{n,k}; (A \cdot B)_{ij} = \sum_{l=1}^{m} A_{il} B_{lj}$

69. Inverzní matice: $\mathbb{R}^{n,n} \to \mathbb{R}^{n,n}; A^{-1} \cdot A = A \cdot A^{-1} = \mathbb{I} = id$

70. Vektorový součin: $\times : V^3 \times V^3 \to V^3; a \times b = \mathcal{E}_{ijk} a_i b_j e_k$

71. Hodnost: $Rank : \mathbb{R}^{n,m} \to \mathbb{N}_0; Rank A = \dim span(A_{.1}, ..., A_{.n})$

72. Defekt: $d : \mathcal{L}(X, Y) \to \mathbb{N}_0; d(A) = \dim \ker(A)$

73. Vnější derivace: $\omega : C^1(\mathbb{R}^n) \to (C^1(\mathbb{R}^n))^{\#}; df(x) = f'(x)$

74. Taylorův rozvoj: $C^{\omega}(\mathbb{R}^n) \to C^{\omega}(\mathbb{R}^n); f(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$

75. Diferenciální forma: $\omega = \sum_{n=1}^{n} \omega_i \underset{\leftarrow}{e}^{i}; \omega_i : \mathbb{R}^n \to \mathbb{R}$

76. Skalární pole: $f : \mathbb{R}^n \to \mathbb{R}$

77. Vektorové pole: $\overrightarrow{F} : \mathbb{R}^n \to (V^n)$

78. Kovektorové pole: $\omega : \mathbb{R}^n \to (V^n)^{\#}$

79. Tenzorové pole: $\overleftrightarrow{T} : \mathbb{R}^n \to \mathscr{L}(V^n, T)$

80. Laplaceova transformace: $\mathscr{L} : \mathscr{S} \to ?; \mathscr{L}[f(x)](\xi) = \int_0^{+\infty} e^{-x\xi} f(x) dx$

81. Standardní skalární součin: $\langle . \mid . \rangle : \mathbb{C}^n \times \mathbb{C}^n \to \mathbb{R}_0^+; \langle x \mid y \rangle = \sum_{i=1}^{n} \overline{x}_i y_j$

82. Vzdálenost množin: $dist : \mathcal{P}(X, \rho) \times \mathcal{P}(X, \rho) \to \mathbb{R}_0^+; dist(A, B) = \inf_{x \in A; y \in B} \rho(x, y)$

83. Klobouk: $\omega_\varepsilon : \mathbb{R} \to [0; C_n] \in \mathscr{D}; \omega_\varepsilon(x) = \varepsilon^{-n} \cdot \begin{cases} C_n e^{\frac{1}{1-|x|^2}}; |x| < 1 \\ 0; |x| \geqslant 1 \end{cases}$

84. Vyhlazovací fce: $\eta_\varepsilon : \mathbb{R} \to [0; 1]; \eta_\varepsilon = \omega_\varepsilon \star \chi(H_{2\varepsilon}; M)$

85. Objem množiny: $\mu : \mathcal{P}(\mathbb{R}^n) \to \mathbb{R}_0^+; V(M) = \mu(M) = \int_M d^n x$

86. Gaußovka: $f : \mathbb{R}^n \to \mathbb{R}_+; f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

87. Tenzor: $T : (V)^N \to T$; multilineární

88. Pravděodobnostní rozdělení: $w : \mathbb{R}^n \to [0;1]; \int_{\mathbb{R}^n} w(x)dx = 1$

89. Střední hodnota: $\langle . \rangle : \{f\} \to \mathbb{R}; \langle x \rangle = \int_{\mathbb{R}^n} xw(x)dx$

90. Střední kvadratická odchylka: $\langle \Delta . \rangle : \{f\} \to \mathbb{R}; \langle \Delta x \rangle = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$

91. Hermitův polynom: $H_n : \mathbb{R} \to \mathbb{R}; H_n = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$

92. Legendreovy polynom: $P_n = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$

93. Legeandrova transformace: $\mathcal{L} : C^2(\mathbb{R}^n); f'' \neq 0 \to C^2(\mathbb{R}^n); \mathcal{L}[f(x)](\xi) = f(\xi) - \sum f(\xi)\frac{\partial f}{\partial x}(\xi)$

94. Laguerův polynom: $L_n : \mathbb{R} \to \mathbb{R}; L_n(x) = \frac{1}{n!} e^x \frac{d^n}{dx^n}(e^{-x}x^n)$

95. Gradient: $grad : C^1(\mathbb{R}^n) \to (C^0(\mathbb{R}^n))^n; (grad\ f(x))_i = \frac{\partial f(x)}{\partial x_1} = \nabla f$

96. Divergence: $div : (C^1(\mathbb{R}^n))^n \to C^0(\mathbb{R}); div \overrightarrow{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = \nabla \cdot \overrightarrow{F}$

97. Rotace: $rot : (C^1(\mathbb{R}^n))^n \to (C^0(\mathbb{R}^n))^n; (rot \overrightarrow{F})_i = \mathcal{E}_{ijk} \frac{\partial}{\partial x_j} F_k = \nabla \times \overrightarrow{F}$

98. Laplace operathor: $\Delta : C^2(\mathbb{R}^n) \to C^0(\mathbb{R}^n); \Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = div\ grad\ f$
$rot\ rot = \Delta - grad\ div$

99. Diracův operátor: $D : C^1(\mathbb{R}^n) \to C^1(\mathbb{R}^n); D = \sqrt{\Delta}$

100. Rekurze: $(a_n)_{n=1}^{+\infty} : \mathbb{N} \to \mathbb{C}; a_1 = a; a_{n+1} = f(a_n)$

101. Akce grupy: $\cdot : G \times X \to X; e \cdot x = x \wedge g_1 g_2 \dot{x} = g_1 \cdot (g_2 \cdot x)$

102. Sférická transformace: $sf : (\mathbb{R}^2 \backslash P_\pi) \times \mathbb{R} \to \mathbb{R}^3; x = r\cos\Theta\cos\varphi; y = r\cos\Theta\sin\varphi; z = r\cos\Theta$

103. Wronckián: $W : C^{(n-1)}(\mathbb{R}) \to \mathbb{R}^{n,n}; (W_{y_1,\ldots,y_n})_{ij} = y_j^{(i-1)}$

104. Cylindrická transformace: $cyl : (\mathbb{R}^2 \backslash P_\pi) \times \mathbb{R} \to \mathbb{R}^3; x = r\cos\varphi; y = r\sin\varphi; z = z$

105. Polární transformace: $pol : (\mathbb{R}^2 \backslash P_\pi) \to \mathbb{R}^2; x = r\cos\varphi; y = r\sin\varphi$

106. Tenzorový součin $\otimes : V \times W \to \mathscr{L}(V,W); (x \otimes y)_{ij} = x_i y_j$

107. Vnější násoení: $\wedge : \Lambda(V^n) \times \Lambda(V^n) \to \Lambda(V^n); (\sigma \wedge \rho)(\overrightarrow{x}_1, ..., \overrightarrow{x}_{k+l}) = \frac{1}{k!l!} \sum_{\pi \in S_{k+l}} sgn\pi\ \sigma(\overrightarrow{x}_{\pi(1)}, ..., \overrightarrow{x}_{\pi(k)})\rho(\overrightarrow{x}_{\pi(k+1)}, ..., \overrightarrow{x}_{\pi(k+l)})$