Problem 9

$$A_\mu = \begin{pmatrix} 0 \\ 0 \\ -Bx \\ 0 \end{pmatrix}$$

$$\Pi_\mu = (i\partial_\mu - qA_\mu) \;;\; \psi_{(\vec{x},t)} = e^{-iEt}\begin{pmatrix} \varphi(\vec{x}) \\ \chi(\vec{x}) \end{pmatrix} \;;\; A^\mu = \begin{pmatrix} 0 \\ 0 \\ Bx \\ 0 \end{pmatrix}$$

$$\boxed{(A\otimes B)\begin{pmatrix} \varphi \\ \chi \end{pmatrix} = A\begin{pmatrix} B\varphi \\ B\chi \end{pmatrix}}$$

$$(\gamma^\mu \Pi_\mu - m\mathbb{I})\,\psi_{(x,t)} = 0$$

$$\left(\gamma^\mu (i\partial_\mu - qA_\mu) - m\mathbb{I}\right) e^{-iEt}\begin{pmatrix} \varphi(x) \\ \chi(x) \end{pmatrix} = 0$$

$$\cancel{i}\gamma^0(-\cancel{i}E)\cancel{e^{-iEt}}\begin{pmatrix} \varphi \\ \chi \end{pmatrix} + i\gamma^j\partial_j\begin{pmatrix} \varphi(x) \\ \chi(x) \end{pmatrix}\cancel{e^{-iEt}} + (+q\gamma^2 Bx - m\mathbb{I})\cancel{e^{-iEt}}\begin{pmatrix} \varphi \\ \chi \end{pmatrix}_2$$

$$E\sigma^3\otimes\mathbb{I}\begin{pmatrix} \varphi \\ \chi \end{pmatrix} - \sigma^2\otimes\sigma^j\partial_j\begin{pmatrix} \varphi \\ \chi \end{pmatrix} + iqBx\,\sigma^2\otimes\sigma^2\begin{pmatrix} \varphi \\ \chi \end{pmatrix} - m\begin{pmatrix} \varphi \\ \chi \end{pmatrix} = 0$$

$$E\sigma^3\begin{pmatrix} \varphi \\ \chi \end{pmatrix} - \sigma^2\begin{pmatrix} \sigma^j\partial_j\varphi \\ \sigma^j\partial_j\chi \end{pmatrix} + iqBx\,\sigma^2\begin{pmatrix} \sigma^2\varphi \\ \sigma^2\chi \end{pmatrix} - m\mathbb{I}\begin{pmatrix} \varphi \\ \chi \end{pmatrix} = 0$$

$$E\varphi + i\sigma^j\partial_j\chi + \overset{+1}{\cancel{i}}qBx\sigma^2\chi - m\varphi = 0$$

$$-E\chi - i\sigma^j\partial_j\varphi + \overset{-}{\cancel{i}}qBx\sigma^2\varphi - m\chi = 0$$

$$(E+m)\chi = -i\sigma^j\partial_j\varphi \blacksquare qBx\sigma^2\varphi$$

$$(E-m)\varphi + i\sigma^j\partial_j\frac{-qBx\sigma^2 - i\sigma^k\partial_k\varphi}{E+m} + qBx\sigma^2\frac{-qBx\sigma^2 - i\sigma^j\partial_j}{E+m}\varphi = 0$$

$$\blacksquare iqB\,\sigma^1\sigma^2\varphi$$

$$(E^2-m^2)\varphi \blacksquare iqBx\,\sigma^j\sigma^2\partial_j\varphi + \sigma^j\sigma^k\underset{\text{sym}}{\partial_j\partial_k}\varphi - q^2B^2x^2\sigma^2\sigma^2\varphi \blacksquare iqBx\,\sigma^2\sigma^j\partial_j\varphi = 0$$

$$\blacksquare iqB\sigma^3\varphi$$

$$(E^2-m^2)\varphi \blacksquare iqBx\left(\partial_2\varphi \cancel{+i\varepsilon^{j2k}}\partial_k\varphi\right) + \overset{\Delta}{\partial_k^2}\varphi - q^2B^2x^2\varphi \blacksquare iqBx\left(\partial_2\varphi \cancel{+i\overset{-\varepsilon^{j2k}}{\overset{\shortparallel}{\varepsilon^{2jk}}}}\partial_k\varphi\right) = 0$$

$$+qB\sigma^3\varphi$$

$$(E^2-m^2)\varphi \blacksquare 2iqBx\,\partial_2\varphi + \Delta\varphi - q^2B^2x^2\varphi = 0$$

$$(E^2-m^2)\varphi = -\Delta\varphi + 2iqBx\,\partial_2\varphi + q^2B^2x^2\varphi \blacksquare qB\sigma^3\varphi = \hat{p}^2\varphi \blacksquare 2qBx\,\hat{p}_y\varphi + q^2x^2B^2\varphi - qB\sigma^3\varphi$$

$\hat{p}_x = -i\partial_x$; $\hat{p}^2 = p_x^2 + p_y^2 + p_z^2 = -\Delta$

$$(E^2 - m^2)\varphi = (-2qBx\hat{p}_y + \hat{p}^2 + q^2B^2x^2)\varphi$$

$-qB\sigma^3$

$$(E^2 - m^2)\varphi = (-2qBx\hat{p}_y + \hat{p}^2 - qB\sigma^3 + q^2B^2x^2)\varphi$$