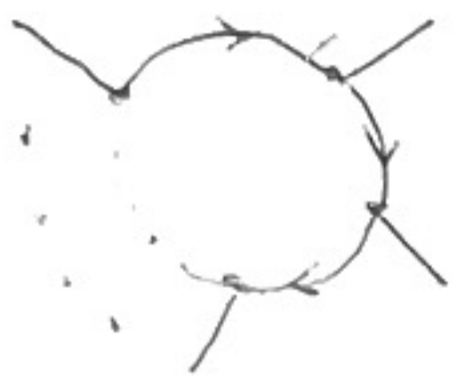
15.4.2020.

CV

CONSIDER YUKAWA INTERACTING TERM
$\mathcal{L}_{INT} = -g\bar{\psi}\psi\phi$ AND SHOW THAT THE LOOP
DIAGRAM (THAT MIGHT BE PART OF A
BIGGER DIAGRAM)



N - VERTICES
N - PROPAGATORS

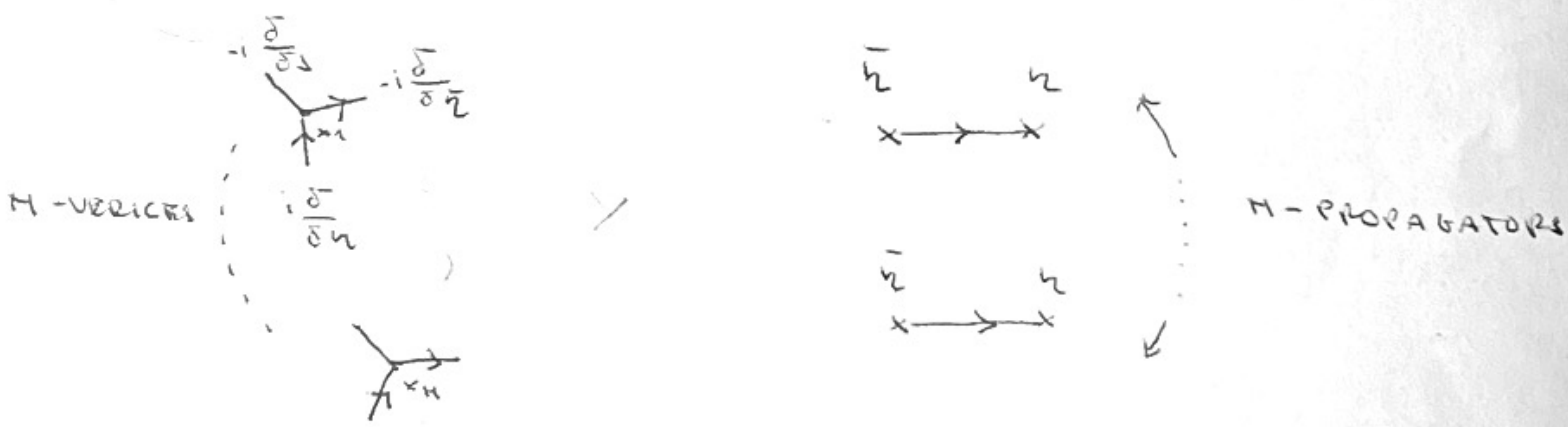
HAS ALWAYS OPOSITE SIGN IN COMPARISON
WITH ANALOGOU LOOP DIAGRAM IN SCALAR
THEORY (<u>SCALAR YUKAWA</u> INTERACTION TERM
$\mathcal{L}_{INT} = -g\,\phi^{\dagger}\phi\,\varphi$).

<u>HINT:</u>

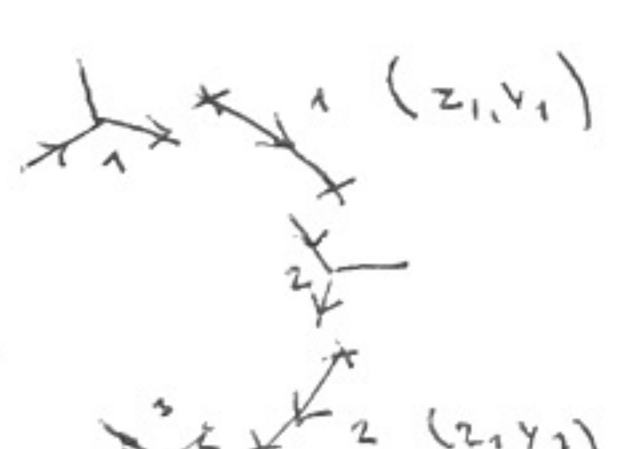
ABOVE DIAGRAM IS OBTAINED BY FROM

$$\prod_{i=1}^{N}\left(i\frac{\delta}{\delta\eta(x_i)}\right)\left(-i\frac{\delta}{\delta\bar{\eta}(x_i)}\right)\prod_{j=1}^{N}\left[\int d^4z_j\,d^4y_j\,\bar{\eta}(z_j)\,iS_F(z_j-y_j)\,\eta(y_j)\right]$$

(PART WITH BOSONIC SOURCES IS DECOUPLED)



$\Downarrow$



(THIS CAN BE ARBITRARILY
RESHUFFLED, BECAUSE
$\frac{\delta}{\delta\eta_i}\frac{\delta}{\delta\bar{\eta}_i}$ and $\frac{\delta}{\delta\eta_j}\frac{\delta}{\delta\bar{\eta}_j}$
COMMUTE.

$\partial\eta(x_1)\partial\bar\eta(x_1)\partial\eta(x_2)\partial\bar\eta(x_2)\dots\partial\eta(x_n)\partial\bar\eta(x_n)\big(\bar\eta(z_1)\eta(y_1)\bar\eta(z_2)\eta(y_2)\dots\bar\eta(z_n)\eta(y_n)\big)$

i) přiřadíme $\partial\bar\eta(x_i) \sim \bar\eta(z_i)$, $\forall i \in \hat{n}$

znaménko : $(-1)^{2(n-1)}\cdot(-1)^{2(n-2)+1}\cdot(-1)^{2(n-3)+2}\cdot\ldots\cdot(-1)^{2\cdot 0+(n-1)} = (-1)^\alpha$

$$\alpha = \sum_{k=0}^{n-1} 2k + \sum_{k=0}^{n-1} k = \frac{3(n-1)n}{2}$$

máme : $(-1)^{\frac{3(n-1)n}{2}} \partial\eta(x_1)\partial\eta(x_2)\dots\partial\eta(x_{n-1})\partial\eta(x_n)\big(\eta(x_1)\eta(x_2)\dots\eta(x_{n-1})\eta(x_n)\big)$

ii) následně přiřadíme · $\partial\eta(x_i) \sim \eta(z_{i+1})$, $\forall i \in \{1,\dots,n-1\}$ a)
· $\partial\eta(x_n) \sim \eta(x_n)$ b)

znaménko a) : $(-1)^{(n-1)+1}\cdot(-1)^{(n-2)+1}\cdot(-1)^{(n-3)+1}\dots(-1)^{1+1} = (-1)^\beta$
znaménko b) : $(+1)$

$$\beta = \sum_{k=2}^{n} k = \sum_{k=1}^{n} k - 1 = \frac{n(n+1)}{2} - 1$$

Celkové znaménko : $2(n+1)n - 1 = \gamma$

$$(-1)^{2(n-1)n-1} = -1 \,, \forall n \in \mathbb{N}$$

$\exp\left(-i\int d^4x \left(-i\frac{\partial}{\partial\bar\eta(x)}\right)\left(+i\frac{\partial}{\partial\eta(x)}\right)\left(i\frac{\partial}{\partial J(x)}\right)\right)\exp\left(i\int d^4z\,d^4y\,\bar\eta(z)\,iS_F(z-y)\,\eta(y)\right) \sim$

zajímá nás pouze uzavřený diagram

$\sim (\dots) + (\dots)\prod_{i=1}^{n}\left(i\frac{\partial}{\partial\eta(x_i)}\right)\left(-i\frac{\partial}{\partial\bar\eta(x_i)}\right)\prod_{j=1}^{n}\left[\int d^4z_j\,d^4y_j\,\bar\eta(z_j)\,iS_F(z_j-y_j)\,\eta(y_j)\right]$

$$\frac{3(n-1)n}{2} + \frac{n(n+1)}{2} - 1 =$$

$$= \frac{1}{2}\left(3n^2 - 3n + n^2 + n - 2\right) = \frac{1}{2}\left(4n^2 + 2n - 2\right) = \left(2n^2 + n - 1\right) =$$

$$= n(2n+1) - 1$$

n sudé: $n = 2k$: $\overbrace{2k(4k+1)}^{2(\dots)} - 1$ $\rightarrow$ liché ✓

n liché: $n = 2k+1$: $\underbrace{(2k+1)(2k+2)}_{2(\dots)(\dots)} - 1$ $\rightarrow$ liché