QFT2: Problem 22

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Find a expansion of $\langle 0| T[\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\phi(x_5)\phi(x_6)] |0\rangle$
Short form: $< x_1 x_2 x_3 x_4 x_5 x_6 >$
In therms of propagator $\langle 0| T[\phi(x)\phi(y)] |0\rangle$ and represent result graphically.

We take our given expression and write it as generated same as the following formula / equality.

$$\langle 0| T[e^{\int d^4x K(x)\phi(x)}] |0\rangle = 1 * e^{\frac{1}{2}\int d^4x d^4y K(x) \langle 0|T[\phi(x)\phi(y)]|0\rangle K(y)} \tag{1}$$

$$\text{where } \langle 0| T[\phi(x)\phi(y)] |0\rangle = i\Delta_F(x-y)$$

Functional deviate will give us (2) and then apply Wick's theorem will give (3).

$$\frac{\delta}{\delta K(x_1)} \cdots \frac{\delta}{\delta K(x_6)} \langle 0| T[e^{\int d^4x K(x)\phi(x)}] |0\rangle \Big|_{K\equiv 0} \tag{2}$$

$$\frac{\delta}{\delta K(x_1)} \cdots \frac{\delta}{\delta K(x_6)} e^{\frac{1}{2}\int d^4x d^4y K(x)\langle 0|T[\phi(x)\phi(y)]|0\rangle K(y)} \Big|_{K\equiv 0} \tag{3}$$

Since K = 0, there will be only 1 non-vanishing point eg. Only 3rd expansion will contribute, therefore.

$$\frac{\delta^6}{\delta K(x_1)...\delta K(x_6)} \frac{1}{3!}\frac{1}{8} \int d^4y_1...d^4y_6 K(y_1)...K(y_6) \langle 0| T[\phi(y_1)\phi(y_2)] |0\rangle \langle 0| T[\phi(y_3)\phi(y_4)] |0\rangle \langle 0| T[\phi(y_5)\phi(y_6)] |0\rangle \tag{4}$$

$$\frac{1}{(6)(8)} \sum_{\sigma\in S_6} \int dy_1...dy_6 \delta(x_{\sigma(1)} - y_1)...\delta(x_{\sigma(6)} - y_6) \langle 0| T[\phi(y_1)\phi(y_2)] |0\rangle \langle 0| T[\phi(y_3)\phi(y_4)] |0\rangle \langle 0| T[\phi(y_5)\phi(y_6)] |0\rangle \tag{5}$$

$$\frac{1}{48} \sum_{\sigma\in S_6} \langle 0| T[\phi(x_{\sigma(1)})\phi(x_{\sigma(2)})] |0\rangle \langle 0| T[\phi(x_{\sigma(3)}\phi(x_{\sigma(4)})] |0\rangle \langle 0| T[\phi(x_{\sigma(5)})\phi(x_{\sigma(6)})] |0\rangle \tag{6}$$

there us 6! = 720 Terms.. → but only (5)(3) = 15 are different.

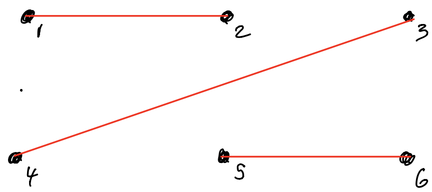
From this point short notation will be used.

possible contractions logic =

$(< x_1x_2 >) * (< x_3x_4 >< x_5x_6 > + < x_3x_5 >< x_4x_6 > + < x_4x_5 >< x_3x_6 >) +$
$(< x_1x_3 >) * (< x_2x_4 >< x_5x_6 > + < x_2x_5 >< x_4x_6 > + < x_2x_6 >< x_4x_5 >) +$
$(< x_1x_4 >) * (< x_2x_3 >< x_5x_6 > + < x_2x_5 >< x_3x_6 > + < x_2x_6 >< x_3x_5 >) +$
$(< x_1x_5 >) * (< x_2x_3 >< x_4x_6 > + < x_2x_4 >< x_3x_6 > + < x_2x_6 >< x_3x_4 >) +$
$(< x_1x_6 >) * (< x_2x_3 >< x_4x_5 > + < x_2x_4 >< x_3x_5 > + < x_2x_5 >< x_3x_4 >)$

$$
\begin{aligned}
< x_1x_2x_3x_4x_5x_6 > = & < x_1x_2 >< x_3x_4 >< x_5x_6 > + < x_1x_6 >< x_2x_3 >< x_4x_5 > \\
& + < x_1x_4 >< x_3x_5 >< x_2x_6 > + < x_1x_5 >< x_3x_6 >< x_2x_4 > \\
& + < x_1x_3 >< x_4x_6 >< x_2x_5 > + < x_1x_2 >< x_4x_5 >< x_3x_6 > \\
& + < x_1x_2 >< x_3x_5 >< x_4x_6 > + < x_1x_4 >< x_2x_3 >< x_5x_6 > \\
& + < x_1x_5 >< x_2x_3 >< x_4x_6 > + < x_1x_5 >< x_2x_6 >< x_3x_4 > \\
& + < x_1x_6 >< x_3x_4 >< x_2x_5 > + < x_1x_3 >< x_4x_5 >< x_2x_6 > \\
& + < x_1x_3 >< x_2x_4 >< x_5x_6 > + < x_1x_6 >< x_2x_4 >< x_3x_5 > \\
& + < x_1x_4 >< x_2x_5 >< x_3x_6 >
\end{aligned}
$$

(1)



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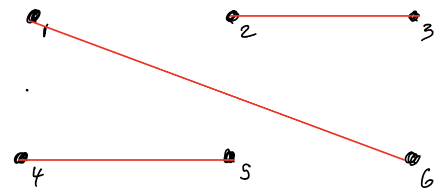

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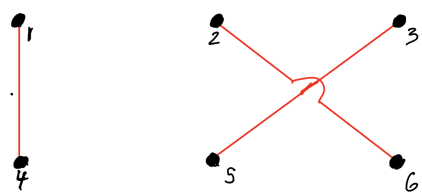

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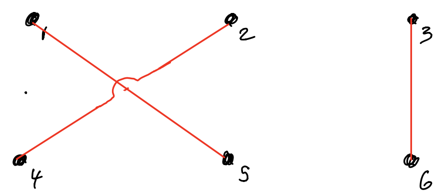

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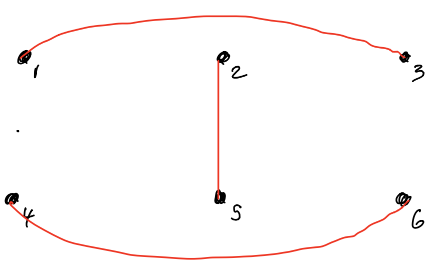

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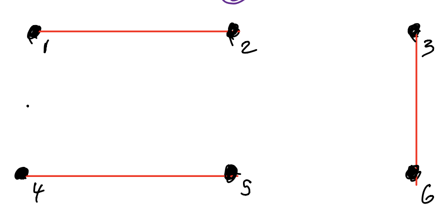

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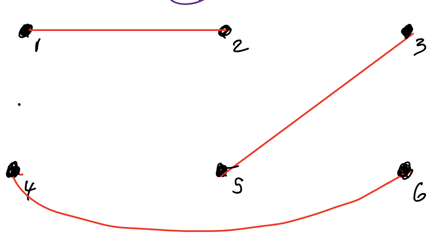

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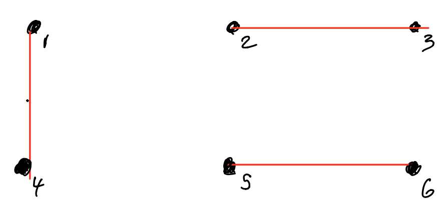

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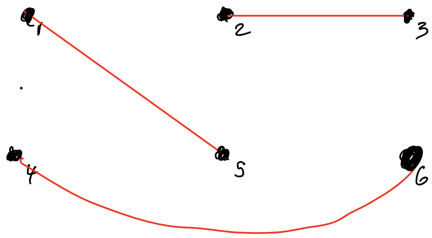

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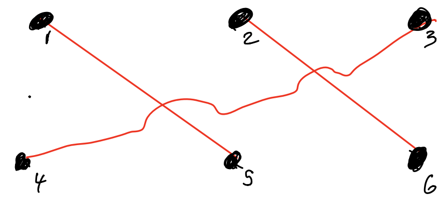

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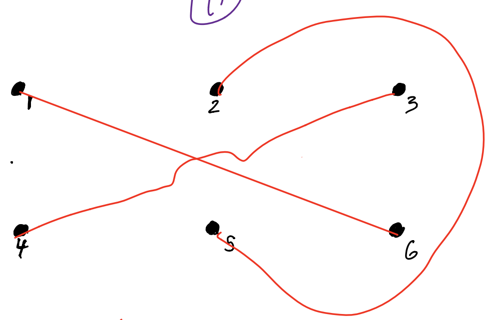

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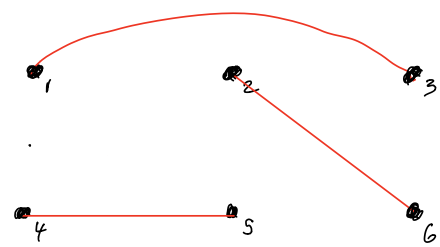

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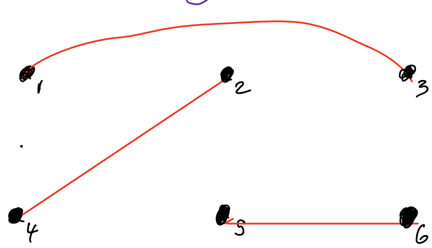

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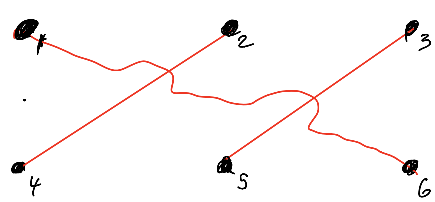

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