Problem 25

Problem 25: *Determine the 1-point and 2-point Green function of the ϕ^3- theory (Eq. (243) in "Tutorials" document) up to order g^2.*

In this task we will use many of the results of previous exercises and problems. From problem 24 we have $\tilde{Z}[J]$ represented diagrammatically (with "diagram dictionary") up to order g^2. The only thing we have to do with this result is to convert the diagrams to formulas and write the expansion of $\tilde{Z}_{g=0}[J] = \exp[-\frac{i}{2}\int d^4x'd^4yJ(x')\triangle_F(x'-y)J(y)]$:

$$\tilde{Z}[J] \approx \{1 + \frac{ig}{6}\int d^4x \left[3i\triangle_F(0)\int d^4y\triangle_F(x-y)J(y) + ...\right] -$$

$$-\frac{g^2}{72}\int d^4xd^4z[-18(\triangle_F(x-z))^2\left(\int d^4y\triangle_F(x-y)J(y)\right)\left(\int d^4y'\triangle_F(z-y')J(y')\right)$$

$$-9\triangle_F(0)\triangle_F(x-z)\left(\int d^4y\triangle_F(x-y)J(y)\right)^2 - 9\triangle_F(0)\triangle_F(x-z)\left(\int d^4y\triangle_F(z-y)J(y)\right)^2 -$$

$$-9\triangle_F(0)^2\left(\int d^4y\triangle_F(x-y)J(y)\right)\left(\int d^4y'\triangle_F(z-y')J(y')\right) + ...]\}\cdot$$

$$\cdot\left(1 - \frac{i}{2}\int d^4yd^4zJ(y)\triangle_F(y-z)J(z) + ...\right), \qquad (1)$$

where "..." means terms in which "J^3" and higher orders occur. Now lets look at 1-point Green function. Here we need terms in which J is in 1st power so only this terms will be written.

$$\langle x_1\rangle = -i\frac{\delta}{\delta J(x_1)}\tilde{Z}\mid_{J=0}[J] = -i\frac{\delta}{\delta J(x_1)}\left[-\frac{g}{2}\int d^4x\triangle_F(0)\int d^4y\triangle_F(x-y)J(y)\right] =$$

$$\frac{g}{2}\int d^4x\triangle_F(x-x_1)i\triangle_F(0), \qquad (2)$$

(Also $|_{J=0}$ was omitted, becouse J vanisch in final expression.) This is the same result as we got in exercise 94. Diagramatic representation is on figure 1.

Another task is the 2-point Green function, so the relavant terms from (1) are those that have "J^2" (again other terms and $|_{J=0}$ omitted) :

$$\langle x_1, x_2\rangle = (-i)^2\frac{\delta}{\delta J(x_2)}\frac{\delta}{\delta J(x_1)}\tilde{Z}\mid_{J=0}[J] = (-i)^2\frac{\delta}{\delta J(x_2)}\frac{\delta}{\delta J(x_1)}[-\frac{i}{2}\int d^4yd^4zJ(y)\triangle_F(y-z)J(z)$$

$$-\frac{g^2}{72}\int d^4xd^4z[-18(\triangle_F(x-z))^2\left(\int d^4y\triangle_F(x-y)J(y)\right)\left(\int d^4y'\triangle_F(z-y')J(y')\right)$$

$$-9\triangle_F(0)\triangle_F(x-z)\left(\int d^4y\triangle_F(x-y)J(y)\right)^2 - 9\triangle_F(0)\triangle_F(x-z)\left(\int d^4y\triangle_F(z-y)J(y)\right)^2 -$$

$$-9\,\triangle_F\,(0)^2\left(\int d^4y\,\triangle_F\,(x-y)J(y)\right)\left(\int d^4y'\,\triangle_F\,(z-y')J(y')\right)] \quad (3)$$

We will do the procedure with each term(s) separately (Due to integration, there are two similar terms so they are both in green). Lets start with the red ones (in fact, we have already seen this in exercise 93):

$$(-i)^2\frac{\delta}{\delta J(x_2)}\frac{\delta}{\delta J(x_1)}[red]=\frac{i}{2}\int d^4xd^4z\triangle_F(y-z)[\delta(y-x_1)\delta(z-x_2)+\delta(z-x_1)\delta(y-x_2)]=$$

$$=\frac{i}{2}[\triangle_F(x_1-x_2)+\triangle_F(x_2-x_1)]=i\,\triangle_F\,(x_1-x_2), \quad (4)$$

Blue terms:

$$(-i)^2\frac{\delta}{\delta J(x_2)}\frac{\delta}{\delta J(x_1)}\left[-\frac{g^2}{72}\int d^4xd^4z\,blue\right]=$$

$$-\frac{g^2}{4}\frac{\delta}{\delta J(x_2)}\int d^4xd^4z(\triangle_F(x-z))^2[\triangle_F(x-x_1)\left(\int d^4y'\,\triangle_F\,(z-y')J(y')\right)+$$

$$+\left(\int d^4y\,\triangle_F\,(x-y)J(y)\right)\triangle_F(z-x_1)]=-\frac{g^2}{4}\int d^4xd^4z(\triangle_F(x-z))^2[\triangle_F(x-x^1)\triangle_F(z-x_2)+$$

$$+\triangle_F(x-x_2)\triangle_F(z-x_1)]=-\frac{g^2}{2}\int d^4xd^4z(\triangle_F(x-z))^2\triangle_F(x-x_1)\triangle_F(z-x_2) \quad (5)$$

Green terms:

$$(-i)^2\frac{\delta}{\delta J(x_2)}\frac{\delta}{\delta J(x_1)}\left[-\frac{g^2}{72}\int d^4xd^4z\,green\right]=$$

$$=-\frac{g^2}{4}\frac{\delta}{\delta J(x_2)}\int d^4xd^4z\triangle_F(0)\triangle_F(x-z)[\triangle_F(x-x^1)\left(\int d^4y\,\triangle_F\,(x-y)J(y)\right)+$$

$$+\triangle_F(z-x_1)\left(\int d^4y\,\triangle_F\,(z-y)J(y)\right)]=-\frac{g^2}{4}\int d^4xd^4z\triangle_F(0)\triangle_F(x-z)[\triangle_F(x-x_1)\triangle_F(x-x_2)+$$

$$+\triangle_F(z-x_1)\triangle_F(z-x_2)]=-\frac{g^2}{2}\int d^4xd^4z\triangle_F(0)\triangle_F(x-z)\triangle_F(x-x_1)\triangle_F(x-x_2) \quad (6)$$

Purple terms:

$$(-i)^2\frac{\delta}{\delta J(x_2)}\frac{\delta}{\delta J(x_1)}\left[-\frac{g^2}{72}\int d^4xd^4z\,purple\right]=$$

$$=-\frac{g^2}{8}\frac{\delta}{\delta J(x_2)}\int d^4xd^4z\,\triangle_F\,(0)^2[\triangle_F(x-x_1)\left(\int d^4y'\,\triangle_F\,(z-y')J(y')\right)+$$

$$+\left(\int d^4y\,\triangle_F\,(x-y)J(y)\right)\triangle_F(z-x_1)]=-\frac{g^2}{8}\int d^4xd^4z\triangle_F(0)^2[\triangle_F(x-x_1)\triangle_F(z-x_2)+$$

$$+\triangle_F(x-x_2)\triangle_F(z-x_1)] = -\frac{g^2}{4}\int d^4x d^4z \triangle_F(0)^2 \triangle_F(x-x_1)\triangle_F(z-x_2) \quad (7)$$

That's all we had to compute. We should now compare the result with the result of exercise 94 (same task, different procedure). We actually obtained one term with the 1st power of g with correct pre-factor 1 (symmetry factor $1/S$ in the exercise) and three terms with g^2 in which the correct pre-factors occur $(1/2, 1/2, 1/4)$. However, in order to connect our expresions with exercise 94, we need a diagramatic representation. They are schown in figure 1.

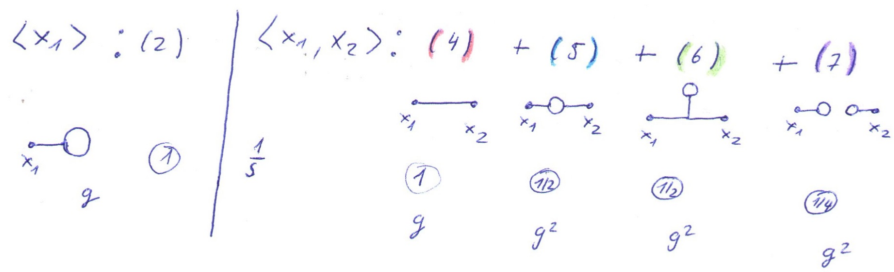

Figure 1: Diagramatic representation of 1-point and 2-point Green function