KTP problem 29.

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Problem 29. Based on the result

$$f(z_0) = \frac{1}{\pi}\int_c^{+\infty} \frac{\mathrm{Im}(f(s+i\varepsilon))}{s-z_0}\mathrm{d}s + \sum_{k=1}^{n} \frac{\mathrm{Res}(f;z_k)}{z_0-z_k}$$

find the wave-function renormalization factor Z andthe spectral function $\sigma(M^2)$ in terms of the momentum-space propagator $\tau(p)$.

According to formula for $\tau(p)$, we can take $\tau \equiv \tau(p^2)$, where we extend p^2 up to the complex plane.

$$\tau(p^2) = \frac{iZ}{p^2-m^2+i\varepsilon} + i\int_{M_t^2}^{+\infty} \mathrm{d}(M^2)\sigma(M^2)\frac{1}{p^2-M^2+i\varepsilon}$$

When we take $f(z_0)$ and put $z_0 = p^2 + i\varepsilon$, $s = M^2$, $z_k = m_k^2$ and for the simplicity we assume case where there are no bound states before branch cut that starts with $c = M_t^2$ then we obtain

$$f(p^2+i\varepsilon) = \frac{1}{\pi}\int_{M_t^2}^{+\infty} \frac{\mathrm{Im}(f(M^2+i\varepsilon))}{M^2-p^2-i\varepsilon}\mathrm{d}(M^2) + \sum_{k=1}^{n} \frac{\mathrm{Res}(f;m_k^2)}{p^2+i\varepsilon-m_k^2}$$

then

$$i\tau(p^2) = \int_{M_t^2}^{+\infty} \frac{\sigma(M^2)}{M^2-p^2-i\varepsilon}\mathrm{d}(M^2) + \frac{-Z}{p^2-m^2+i\varepsilon}.$$

When we compare $f(p^2+i\varepsilon)$ and $i\tau(p^2)$

$$f(p^2+i\varepsilon) = i\tau(p^2)$$

$$\frac{1}{\pi}\int_{M_t^2}^{+\infty} \frac{\mathrm{Im}(i\tau(p^2=M^2))}{M^2-p^2-i\varepsilon}\mathrm{d}(M^2) + \sum_{k=1}^{n} \frac{\mathrm{Res}(i\tau;m_k^2)}{p^2-m_k^2+i\varepsilon} = \int_{M_t^2}^{+\infty} \frac{\sigma(M^2)}{M^2-p^2-i\varepsilon}\mathrm{d}(M^2) + \frac{-Z}{p^2-m^2+i\varepsilon}$$

we get the result for wave function renormalization factor

$$Z = -\mathrm{Res}(i\tau(p^2=m^2))$$

and for spectral function

$$\sigma(M^2) = \frac{\mathrm{Im}(i\tau(p^2=M^2))}{\pi}.$$