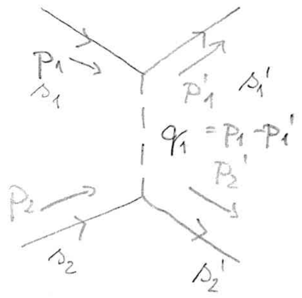
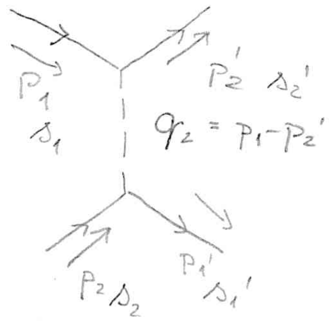
Problem 31

Scattering of 2 nucleons $n + n \to n + n$ with $\mathcal{L}_I(\psi, \bar\psi, \Phi) = -g\bar\psi\psi\Phi$ (ψ is a Dirac fermion field with mass m discribing nucleon, Φ is a real scalar field with mass M.) In order g^2 calculate $\sum_s |T_{fi}|^2$ in the CMS.

- přispívají 2 diagramy:





$$T_{fi} = -ig^2\left[\frac{\bar u(p_1', s_1')u(p_1, s_1)\bar u(p_2', s_2')u(p_2, s_2)}{(p_1 - p_1')^2 - M^2} - \frac{\bar u(p_2', s_2')u(p_1, s_1)\bar u(p_1', s_1')u(p_2, s_2)}{(p_1 - p_2')^2 - M^2}\right]$$

- ozn.: $T_1 = -ig^2 \dfrac{\bar u(p_1', s_1')u(p_1, s_1)\bar u(p_2', s_2')u(p_2, s_2)}{(p_1 - p_1')^2 - M^2}$ $\quad T_2 = ig^2 \dfrac{\bar u(p_2', s_2')u(p_1, s_1)\bar u(p_1', s_1')u(p_2, s_2)}{(p_1 - p_2')^2 - M^2}$

$$\Rightarrow \sum_{s_i} |T_{fi}|^2 = \sum_s T_{fi}T_{fi}^* = \sum_{s_i}\left(|T_1|^2 + |T_2|^2 + T_1 T_2^\dagger + T_2 T_1^\dagger\right)$$

- pro pozdější zjednodušení použijeme Mandelstamovy proměnné

$$s = (p_1 + p_2)^2 = (p_1' + p_2')^2 \overset{CMS}{\Rightarrow} (p_1 \cdot p_2) = (p_1' \cdot p_2') = -m^2 + \frac{s}{2}$$

$$t = (p_1 - p_1')^2 = (p_2 - p_2')^2 \overset{CMS}{\Rightarrow} (p_1 \cdot p_1') = (p_2 \cdot p_2') = m^2 - \frac{t}{2}$$

$$u = (p_1 - p_2')^2 = (p_2 - p_1')^2 \overset{CMS}{\Rightarrow} (p_1 \cdot p_2') = (p_2 \cdot p_1') = m^2 - \frac{u}{2}$$

- spočítáme jednotlivé členy $\sum_s |T_{fi}|^2$

(1) $$\sum_s |T_1|^2 = \sum_s \frac{g^4}{((p_1 - p_1')^2 - M^2)^2}\left[\bar u(p_1', s_1')u(p_1, s_1)\underbrace{\bar u(p_2', s_2')\underbrace{u(p_2, s_2)\bar u(p_2, s_2)}_{\not p_2 + m}u(p_2', s_2')}\bar u(p_1, s_1)u(p_1', s_1')\right] =$$

$= \Big\|\ \sum_s u(p, s)\bar u(p, s) = \not p + m \qquad Tr(\not p_i \not p_j) = 4(p_i \cdot p_j) \qquad Tr(\not p_i) = 0 \qquad Tr(m^2 \cdot \mathbb{I}) = 4m^2$

$$\left|\bar u(\,)u = Tr[u\bar u(\,)]\right| \Rightarrow \sum_s \frac{g^4}{((t) - M^2)^2}\left[\bar u(p_1', s_1')\underbrace{u(p_1, s_1)\, Tr\Big[\underbrace{u(p_2', s_2')\bar u(p_2', s_2')}_{\not p_2' + m}(\not p_2 + m)\Big]\bar u(p_1, s_1)}_{\not p_1 + m}u(p_1', s_1')\right]$$

$$= \frac{g^4}{(t - M^2)^2} Tr\Big[\sum_s u(p_1', s_1')\bar u(p_1', s_1')(\not p_1 + m)\Big] Tr\Big[(\not p_2' + m)(\not p_2 + m)\Big] =$$

$$= \frac{g^4}{(t - M^2)^2} Tr\Big[(\not p_1' + m)(\not p_1 + m)\Big] Tr\Big[(\not p_2' + m)(\not p_2 + m)\Big] = \longrightarrow$$

$$\longrightarrow = \frac{g^4}{(t-M^2)^2} Tr\left[\not{p}_1 \not{p}_1' + \not{p}_1 m + \not{p}_1' m + m^2 \mathbb{I}\right] Tr\left[\not{p}_2 \not{p}_2' + \not{p}_2 m + \not{p}_2' m + m^2 \mathbb{I}\right] =$$

$$= \frac{g^4}{(t-M^2)^2} \left(4(p_1 \cdot p_1') + 4m^2\right)\left(4(p_2 \cdot p_2') + 4m^2\right) = \frac{4g^4}{(t-M^2)^2} (2m^2 - t + 2m^2)(2m^2 - t + 2m^2) =$$

$$= \frac{4g^4}{(t-M^2)^2} (4m^2 - t)^2$$

(2) $$\sum_s |T_2|^2 = \sum_s \frac{g^4}{(p_1-p_2')^2 - M^2} \left[\bar{u}(p_2', s_2') u(p_1, s_1) \bar{u}(p_1', s_1') u(p_2, s_2) \bar{u}(p_2, s_2) u(p_1', s_1') \bar{u}(p_1, s_1) u(p_2', s_2')\right] =$$

$$= \Big\| \text{stejně jako } \sum_s |T_1|^2 \Big\| = \frac{4g^4}{(u-M^2)^2} (4m^2 - u)^2$$

(3) $$\sum_s \left(T_1 T_2^\dagger + T_2 T_1^\dagger\right) = \sum_s \frac{-g^4}{(t-M^2)(u-M^2)} \bar{u}(p_1', s_1') u(p_1, s_1) \bar{u}(p_2', s_2') u(p_2, s_2) \bar{u}(p_2, s_2) u(p_1', s_1') \bar{u}(p_1, s_1) u(p_2', s_2')$$

$$- \frac{g^4}{(t-M^2)(u-M^2)} \bar{u}(p_2', s_2') u(p_1, s_1) \bar{u}(p_1', s_1') u(p_2, s_2) \bar{u}(p_2, s_2) u(p_2', s_2') \bar{u}(p_1, s_1) u(p_1', s_1') =$$

$$= \Big\| \text{použijeme stejný postup jako u (1)} \Big\| =$$

$$= - \frac{g^4}{(t-M^2)(u-M^2)} \left[Tr\left[(\not{p}_1' + m)(\not{p}_1 + m)(\not{p}_2' + m)(\not{p}_2 + m)\right] + Tr\left[(\not{p}_2' + m)(\not{p}_1 + m)(\not{p}_1' + m)(\not{p}_2 + m)\right]\right] =$$

$$= \Big\| Tr(\not{p}_i \not{p}_j \not{p}_k \not{p}_l) = 4\left[(p_i \cdot p_j)(p_k \cdot p_l) - (p_i \cdot p_k)(p_j \cdot p_l) + (p_i \cdot p_l)(p_j \cdot p_k)\right] \Big\| =$$

$$= - \frac{8g^4}{(t-M^2)(u-M^2)} \left[(p_1' \cdot p_1)(p_2' \cdot p_2) - (p_1' \cdot p_2')(p_1 \cdot p_2) + (p_1' \cdot p_2)(p_1 \cdot p_2') + m^2(p_1' \cdot p_1) + m^2(p_1' \cdot p_2') + m^2(p_1' \cdot p_2) + \right.$$
$$\left. + m^2(p_2' \cdot p_2) + m^2(p_1 \cdot p_2) + m^2(p_1 \cdot p_2') + m^4\right] =$$

$$= - \frac{8g^4}{(t-M^2)(u-M^2)} \left[\left(m^2 - \frac{t}{2}\right)^2 - \left(\frac{s}{2} - m^2\right)^2 + \left(m^2 - \frac{u}{2}\right)^2 + 2m^2\left(m^2 - \frac{t}{2}\right) + 2m^2\left(m^2 - \frac{u}{2}\right) + 2m^2\left(\frac{s}{2} - m^2\right) + m^4\right]$$

$$= - \frac{2g^4}{(t-M^2)(u-M^2)} \left[(t-4m^2)^2 + (u-4m^2)^2 - (s-4m^2)^2\right]$$

$$\Rightarrow \sum_s |T_{fi}|^2 = 4g^2 \left[\frac{(t-4m^2)^2}{(t-M^2)^2} + \frac{(u-4m^2)^2}{(u-M^2)^2} - \frac{1}{2} \frac{(t-4m^2)^2 + (u-4m^2)^2 - (s-4m^2)^2}{(t-M^2)(u-M^2)}\right]$$