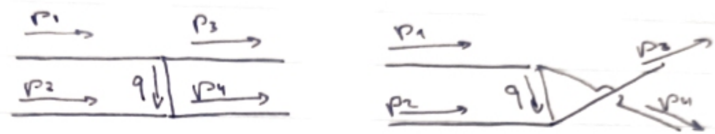
Problém 33:

Feynmanovy diagramy:



$$T_{fi} = (-ig)^2 \frac{i}{(p_1-p_3)^2-m^2} + (-ig)^2 \frac{i}{(p_1-p_4)^2-m^2}$$

$$|T_{fi}|^2 = g^4 \left| \frac{i}{(p_1-p_3)^2-m^2} + \frac{i}{(p_1-p_4)^2-m^2} \right|^2 = g^4 \left[\frac{1}{(p_1-p_3)^2-m^2} + \frac{1}{(p_1-p_4)^2-m^2} \right]^2 =$$

$$= g^4 \left[\frac{(p_1-p_4)^2+(p_1-p_3)^2-2m^2}{[(p_1-p_3)^2-m^2][(p_1-p_4)^2-m^2]} \right]^2$$

- těžišťová soustava: $\vec{p_1} = -\vec{p_2}$ a $\vec{p_3} = -\vec{p_4}$

$$(p_1+p_2)^2 = (p_3+p_4)^2 \rightarrow (E_1+E_2)^2 = (E_3+E_4)^2$$

$$E_1 = \sqrt{m^2+\vec{p_1}^2} = \sqrt{m^2+\vec{p_2}^2} = E_2 \qquad E_3 = \sqrt{M^2+\vec{p_3}^2} = \sqrt{M^2+\vec{p_4}^2} = E_4$$

$$E_1 = E_2 = E_3 = E_4$$

$$E_1 = E_4$$

$$(p_1-p_4)^2 = p_1^2 + p_4^2 - 2p_1\cdot p_4 = E_1^2 - |\vec{p_1}|^2 + E_4^2 - |\vec{p_4}|^2 - 2(E_1E_4 - \vec{p_1}\cdot\vec{p_4}) =$$

$$= -(|\vec{p_1}|^2 + |\vec{p_4}|^2 - 2|\vec{p_1}||\vec{p_4}|\cos\theta)$$

$$(p_1-p_3)^2 = -(|\vec{p_1}|^2 + |\vec{p_3}|^2 - 2|\vec{p_1}||\vec{p_3}|\cos\theta')$$

$$\ldots 2|\vec{p_1}||\vec{p_4}|\cos\theta + |\vec{p_1}|^2 + |\vec{p_3}|^2 - 2|\vec{p_1}||\vec{p_3}|\cos\theta' + 2m^2 \ldots^2$$

$$(p_1-p_3)^2 = -\left(|\vec{p}_1|^2+|\vec{p}_3|^2-2|\vec{p}_1||\vec{p}_3|\cos\theta'\right)$$

$$\rightarrow |T_{fi}|^2 = g^4\left(\frac{|\vec{p}_1|^2+|\vec{p}_4|^2-2|\vec{p}_1||\vec{p}_4|\cos\theta+|\vec{p}_1|^2+|\vec{p}_3|^2-2|\vec{p}_1||\vec{p}_3|\cos\theta'+2m^2}{(|\vec{p}_1|^2+|\vec{p}_4|^2-2|\vec{p}_1||\vec{p}_4|\cos\theta+m^2)(|\vec{p}_1|^2+|\vec{p}_3|^2-2|\vec{p}_1||\vec{p}_3|\cos\theta'+m^2)}\right)^2 =$$

$$\vec{p}_1 = -\vec{p}_2 \quad a \quad \vec{p}_3 = -\vec{p}_4 \rightarrow \theta = \theta' + 180°$$

$$= g^4\left(\frac{|\vec{p}_1|^2+|\vec{p}_3|^2-2|\vec{p}_1||\vec{p}_3|\cos\theta+|\vec{p}_1|^2+|\vec{p}_3|^2+2|\vec{p}_1||\vec{p}_3|\cos\theta+2m^2}{(|\vec{p}_1|^2+|\vec{p}_3|^2-2|\vec{p}_1||\vec{p}_3|\cos\theta+m^2)(|\vec{p}_1|^2+|\vec{p}_3|^2+2|\vec{p}_1||\vec{p}_3|\cos\theta+m^2)}\right)^2 =$$

$$= g^4\left(\frac{2|\vec{p}_1|^2+2|\vec{p}_3|^2+m^2}{(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)^2-(2|\vec{p}_1||\vec{p}_3|\cos\theta)^2}\right)^2$$

$$\rightarrow \frac{d\sigma_{tot}}{d\Omega} = \frac{g^4}{32\pi^2 s}\frac{|\vec{p}_3|}{|\vec{p}_1|}\left(\frac{|\vec{p}_1|^2+|\vec{p}_3|^2+m^2}{(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)^2-(2|\vec{p}_1||\vec{p}_3|\cos\theta)^2}\right)^2$$

$$\sigma_{tot} = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \frac{g^4}{32\pi^2 s}\frac{|\vec{p}_3|}{|\vec{p}_1|}\left[\frac{|\vec{p}_1|^2+|\vec{p}_3|^2+m^2}{(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)^2-(2|\vec{p}_1||\vec{p}_3|\cos\theta)^2}\right]^2 =$$

$$= \frac{g^4}{16\pi s}\frac{|\vec{p}_3|}{|\vec{p}_1|}\int_0^{\pi}\frac{d\theta\sin\theta}{(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)^2}\left[\frac{1}{1-\left(\frac{2|\vec{p}_1||\vec{p}_3|\cos\theta}{|\vec{p}_1|^2+|\vec{p}_3|^2+m^2}\right)^2}\right]^2$$

substituce: $x := \frac{2|\vec{p}_1||\vec{p}_3|}{|\vec{p}_1|^2+|\vec{p}_3|^2+m^2}\cos\theta \quad \rightarrow dx = -\frac{2|\vec{p}_1||\vec{p}_3|}{|\vec{p}_1|^2+|\vec{p}_3|^2+m^2}\sin\theta\, d\theta$

dolní mez integrálu: $\frac{2|\vec{p}_1||\vec{p}_3|}{|\vec{p}_1|^2+|\vec{p}_3|^2+m^2} := C$ horní: $-\frac{2|\vec{p}_1||\vec{p}_3|}{|\vec{p}_1|^2+|\vec{p}_3|^2+m^2}$

$$\to \sigma_{tot} = \frac{g^4}{16\pi s}\,\frac{|\vec{p}_3|}{|\vec{p}_1|}\,\frac{1}{2|\vec{p}_1||\vec{p}_3|\left(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2\right)}\int_c^{-c} - \frac{dx}{(1-x^2)^2} =$$

• integrace: $\displaystyle \operatorname{arctgh}\alpha = \int_0^\alpha \frac{dx}{1-x^2} = \left[\frac{x}{1-x^2}\right]_0^\alpha - \int_0^\alpha \frac{2x^2}{(1-x^2)^2}\,dx =$

$$= \frac{\alpha}{1-\alpha^2} + 2\int_0^\alpha \frac{dx}{1-x^2} - 2\int_0^\alpha \frac{dx}{(1-x^2)^2}$$

$$\to 2\int_0^\alpha \frac{dx}{(1-x^2)^2} = \frac{\alpha}{1-\alpha^2} + \operatorname{arctgh}\alpha$$

$$= \frac{g^4}{16\pi s}\,\frac{|\vec{p}_3|}{|\vec{p}_1|}\left[\frac{\operatorname{arctgh}\left(\frac{2|\vec{p}_1||\vec{p}_3|}{|\vec{p}_1|^2+|\vec{p}_3|^2+m^2}\right)}{2|\vec{p}_1||\vec{p}_3|\left(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2\right)} - \frac{1}{\left(2|\vec{p}_1||\vec{p}_3| - \left(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2\right)^2\right.}\right]$$

• lim − $|\vec{p}_1| \to 0$ ↗ vyraz $\frac{0}{0}$

$$\lim_{|\vec{p}_1|\to 0} \frac{\operatorname{arctgh}\left(\frac{2|\vec{p}_1||\vec{p}_3|}{|\vec{p}_1|^2+|\vec{p}_3|^2+m^2}\right)}{2|\vec{p}_1||\vec{p}_3|\left(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2\right)} \overset{\text{l'Hospitalovo pravidlo}}{=} \lim_{|\vec{p}_1|\to 0} \frac{\dfrac{\left(\frac{2|\vec{p}_1||\vec{p}_3|}{|\vec{p}_1|^2+|\vec{p}_3|^2+m^2}\right)'}{1-\left(\frac{2|\vec{p}_1||\vec{p}_3|}{|\vec{p}_1|^2+|\vec{p}_3|^2+m^2}\right)}}{2|\vec{p}_3|\left(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2\right)+4|\vec{p}_1||\vec{p}_3|} =$$

$$= \lim \frac{\dfrac{\dfrac{2|\vec{p}_3|\left(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2\right)-2|\vec{p}_1||\vec{p}_3|\left(2|\vec{p}_1|\right)}{\left(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2\right)^2}}{\dfrac{\left(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2\right)-2|\vec{p}_1||\vec{p}_3|}{\left(|\vec{p}_1|^2\right)+|\vec{p}_3|^2+m^2)}}}{2|\vec{p}_1||\vec{p}_3|\left(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2\right)+4|\vec{p}_1||\vec{p}_3|} = \lim \frac{+2\left(-|\vec{p}_1|^2+|\vec{p}_3|^2+m^2\right)}{2\left(3|\vec{p}_1|^2+2|\vec{p}_3|^2+2m^2\right)\left(|\vec{p}_1|^2-|\vec{p}_3|^2\right.}$$

$$\lim_{|\vec{p}_1|\to 0} \frac{\ldots(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)}{2|\vec{p}_1||\vec{p}_3|(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)} = \lim_{|\vec{p}_1|\to 0} \frac{1-\left(\frac{2|\vec{p}_1||\vec{p}_3|}{|\vec{p}_1|^2+|\vec{p}_3|^2+m^2}\right)}{2|\vec{p}_3|(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)+4|\vec{p}_1||\vec{p}_3|} =$$

$$= \lim_{|\vec{p}_1|\to 0} \frac{\dfrac{\frac{2|\vec{p}_3|(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)-2|\vec{p}_1||\vec{p}_3|(2|\vec{p}_1|)}{(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)^2}}{\frac{(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)-2|\vec{p}_1||\vec{p}_3|}{(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)}}}{2|\vec{p}_3|(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)+4|\vec{p}_1||\vec{p}_3|} = \lim \frac{+2(-|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)}{2(3|\vec{p}_1|^2+2|\vec{p}_3|^2+2m^2)((|\vec{p}_1|^2-|\vec{p}_3|^2)^2+m^2)} =$$

$$= \frac{1}{(|\vec{p}_3|^2+m^2)}$$

$$\lim_{|\vec{p}_1|\to 0} s = \lim_{|\vec{p}_1|\to 0} (p_1+p_2)^2 = \lim_{|\vec{p}_1|\to 0} \{(E_1+E_2)^2-(\vec{p}_1+\vec{p}_2)^2\} = \lim_{|\vec{p}_1|\to 0} (4E_1^2) = \lim_{|\vec{p}_1|\to 0} 4(m^2+|\vec{p}_1|^2) = 4m^2$$

$$\lim_{|\vec{p}_1|\to 0} \left(-\frac{1}{(2|\vec{p}_1||\vec{p}_3|-(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2))^2}\right) = \frac{1}{(|\vec{p}_3|^2+m^2)^2}$$

$$\rightarrow \lim_{|\vec{p}_1|\to 0} \underbrace{\frac{g^4}{16\pi s}}_{>0} \frac{|\vec{p}_3|}{|\vec{p}_1|} \left[\underbrace{\frac{\operatorname{arctgh}\left(\frac{2|\vec{p}_1||\vec{p}_3|}{|\vec{p}_1|^2+|\vec{p}_3|^2+m^2}\right)}{2|\vec{p}_1||\vec{p}_3|(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)} - \frac{1}{(2(|\vec{p}_1||\vec{p}_3|))^2-(|\vec{p}_1|^2+|\vec{p}_3|^2+m^2)^2}}_{>0} \right] =$$

$$= +\infty$$