Kvantová teorie pole 2 Name: Ondřej Lomický
FJFI ČVUT v Praze Date: 28. 4. 2021

Problem #26

Problem 26: Use Feynman rules to determine the 2-point function $\langle x_1 x_2\rangle$ in the theory with interaction $\lambda\phi^4/4!$ up to order λ^2. (Explicitly calculate the symmetry factors.)
[Hint: Consider only connected diagrams. (Why?)]

The full two-point Green function is

$$\langle x_1 x_2\rangle = -\frac{1}{Z[0]}\frac{\delta^2 Z[J]}{\delta J(x_1)\delta J(x_2)} \tag{1}$$

where $Z[J]$ has following form

$$Z[J] = \exp\left[i\int \mathrm{d}^4 z \mathscr{L}_I\left(-i\frac{\delta}{\delta J(z)}\right)\right]\exp\left[-\frac{i}{2}\int \mathrm{d}^4x_1\mathrm{d}^4x_2 J(x_1)J(x_2)\Delta_F(x_1,x_2)\right] \tag{2}$$

$$Z[J] = \exp\left[-\frac{i\lambda}{4!}\int \mathrm{d}^4 z\left(\frac{\delta}{\delta J(z)}\right)^4\right]\exp\left[-\frac{i}{2}\int \mathrm{d}^4x_1\mathrm{d}^4x_2 J(x_1)J(x_2)\Delta_F(x_1,x_2)\right] \tag{3}$$

where we substituted

$$\mathscr{L}_I(\phi) = -\frac{\lambda}{4!}\phi^4. \tag{4}$$

If we expand the first exponential function into series as follows

$$\exp(x) = \sum_{i=0}^{+\infty}\frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \mathscr{O}(x^3) \tag{5}$$

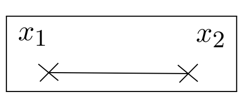
we will get

$$Z[J] = \left[1 - \frac{i\lambda}{4!}\int \mathrm{d}^4 z\left(\frac{\delta}{\delta J(z)}\right)^4 - \frac{\lambda^2}{2\cdot(4!)^2}\int \mathrm{d}^4 z_1\left(\frac{\delta}{\delta J(z_1)}\right)^4\int \mathrm{d}^4 z_2\left(\frac{\delta}{\delta J(z_2)}\right)^4\right]\times$$
$$\times\exp\left[-\frac{i}{2}\int \mathrm{d}^4x_1\mathrm{d}^4x_2 J(x)J(y)\Delta_F(x_1,x_2)\right] \tag{6}$$

The Feynman rules for $\lambda\phi^4/4!$ scalar filed theory in coordinate space are:

- Draw all topologically distinct diagrams. For given n-point Green function with n external legs. For order λ^m use m vertices.
- A line between points x_1 and x_2 represents propagator $i\Delta_F(x_1 - x_2)$.
- A vertex with 4 lines represents a factor $-i\lambda$.
- Integrate over z for all vertices.
- Introduce combinatorial factor $1/S$ where S is the symmetry factor.

For the λ^0:
1)

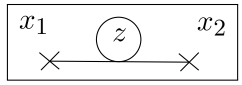


(7)

where is just one possibility. Hence $S = 1$.

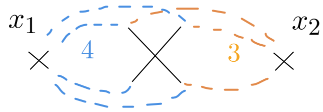
For the λ^1:

1)



(8)

where the multiplicity factor r is $4 \cdot 3 = 12$

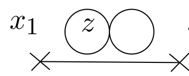


(9)

The symmetry factor S is inverse of the overall pre-factor, i.e.

$$\frac{1}{S} = \frac{4 \cdot 3}{4!} = \frac{1}{2} \Rightarrow S = 2 \tag{10}$$

2)

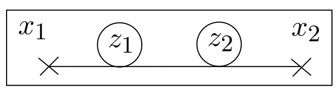


(11)

but this diagram is disconnected.

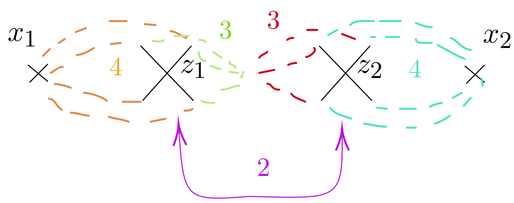
For the λ^2:

1)



(12)

where the multiplicity factor is $4 \cdot 3 \cdot 3 \cdot 4 \cdot 2 = 288$

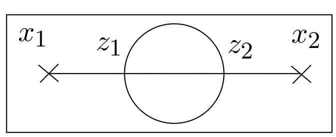


(13)

The symmetry factor is

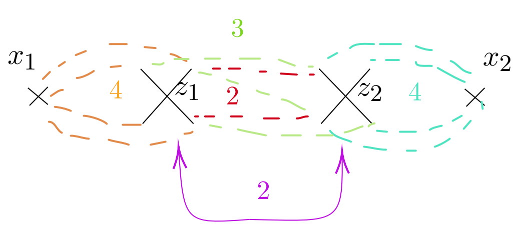
$$\frac{1}{S} = \frac{4 \cdot 3 \cdot 3 \cdot 4 \cdot 2}{2 \cdot (4!)^2} = \frac{1}{4} \Rightarrow S = 4 \tag{14}$$

2)



(15)

where the multiplicity factor is $4 \cdot 3 \cdot 2 \cdot 4 \cdot 2 = 192$

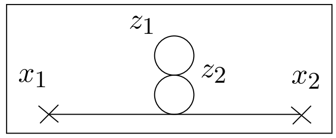


(16)

The related symmetry factor is

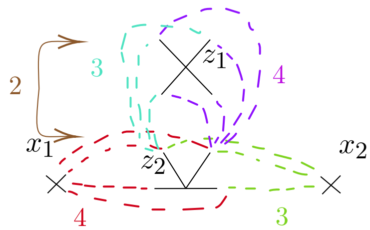
$$\frac{1}{S} = \frac{4 \cdot 3 \cdot 2 \cdot 4 \cdot 2}{\color{red}{2 \cdot (4!)^2}} = \frac{1}{6} \Rightarrow S = 6 \tag{17}$$

3)



(18)

where the multiplicity factor is $4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 = 288$



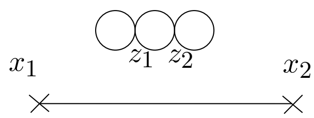
(19)

The related symmetry factor is

$$\frac{1}{S} = \frac{4 \cdot 4 \cdot 3 \cdot 3 \cdot 2}{\color{red}{2 \cdot (4!)^2}} = \frac{1}{4} \Rightarrow S = 4 \tag{20}$$

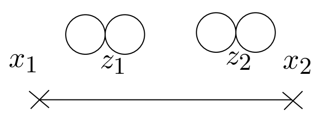
All the other diagrams are disconnected:

5)



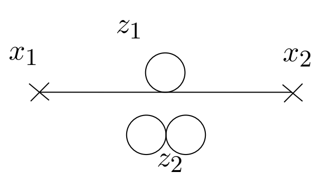
(21)

6)



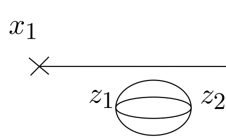
(22)

7)



(23)

8)



(24)

The vacuum diagrams canceled with Z[0] in the denominator in eq. (1). This is known as linked cluster theorem. The connected diagrams are more important for generating the more complicated perturbative diagrams and for scattering purposes in particle physics.