

$$\tilde{Z}[J] = \frac{Z[J]}{Z[0]} \text{ of } \phi^3 \text{ theory up to } g^2.$$
$$1) \quad \tilde{Z}[J] = \frac{Z[J]}{Z[0]}$$

$$(3) \quad \left(-i \frac{\delta}{\delta J(x)}\right)^3 Z_{g=0}[J] = - \left[ 3i \text{ (loop with one external line)} + \text{ (Y-junction with three external lines)} \right] Z_{g=0}[J],$$

where  $Z_{g=0}[J] = \exp\left[-\frac{i}{2} \int d^4x' d^4y J(x') J(y) \Delta_F(x' - y)\right]$  (i.e.  $Z_{g=0}[0] = 1$  and consequently  $\tilde{Z}_{g=0}[J] = Z_{g=0}[J]$ ). The meaning of the individual diagrams can be found at the end of this document.

$$(5) \quad Z[J] \approx \left\{ 1 + i \frac{g}{6} \int d^4 x \left[ 3i \text{ (red tadpole)} + \text{ (red vertex)} \right] - \frac{g^2}{72} \int d^4 x d^4 z \left[ -6i \text{ (blue bubble)} - 9i \text{ (blue chain)} - 18 \text{ (red bubble)} - \right. \right. \\ \left. \left. 9 \text{ (red chain)} - 9 \text{ (red chain)} - 9 \text{ (red chain)} + 3i \text{ (red vertex)} + 3i \text{ (red vertex)} \text{ (red bubble)} + 3i \text{ (red bubble)} \text{ (red vertex)} + \right. \right. \\ \left. \left. \text{ (red vertex)} \text{ (red vertex)} \right] \right\} Z_{g=0}[J].$$
$$(6) \quad Z[0] \approx \left\{ 1 - \frac{g^2}{72} \int d^4x d^4z \left[ -6i \text{ (self-energy diagram)} - 9i \text{ (tadpole diagram)} \right] \right\} \underbrace{Z_{g=0}[0]}_{=1}.$$
$$(7) \quad \tilde{Z}[J] \approx \frac{1+i\frac{g}{6} \int d^4x [\dots] - \frac{g^2}{72} \int d^4x d^4z \left[ -6i \text{ (diagram 1)} - 9i \text{ (diagram 2)} - \dots \right]}{1 - \frac{g^2}{72} \int d^4x d^4z \left[ -6i \text{ (diagram 1)} - 9i \text{ (diagram 2)} \right]} \underbrace{\tilde{Z}_{g=0}[J]}_{=Z_{g=0}[J]},$$

Now we can further approximate using  $\frac{1}{1-x} \approx 1 + x + O(x^2)$

$$(8) \quad \tilde{Z}[J] \approx \left\{ 1 + i\frac{g}{6} \int d^4x [\dots] - \frac{g^2}{72} \int d^4x d^4z \left[ -6i \text{ (diagram)} - 9i \text{ (diagram)} - \dots \right] \right\} \left\{ 1 + \frac{g^2}{72} \int d^4x d^4z \left[ -6i \text{ (diagram)} - 9i \text{ (diagram)} \right] \right\} \tilde{Z}_{g=0}[J].$$

So by only taking terms up to 2<sup>nd</sup> order in  $g$ , the blue terms will cancel out and finally we have

$$(9) \quad \tilde{Z}[J] \approx \left\{ 1 + i\frac{g}{6} \int d^4x \left[ 3i \text{ (diagram)} + \text{ (diagram)} \right] - \frac{g^2}{72} \int d^4x d^4z \left[ -18 \text{ (diagram)} - 9 \text{ (diagram)} - 9 \text{ (diagram)} - 9 \text{ (diagram)} + 3i \text{ (diagram)} + 3i \text{ (diagram)} + 3i \text{ (diagram)} + 3i \text{ (diagram)} + \text{ (diagram)} + \text{ (diagram)} \right] \right\} \tilde{Z}_{g=0}[J].$$

### List of diagrams<sup>1</sup>

$$(a) \quad \text{ (diagram) } = \Delta_F(0) \int d^4y \Delta_F(x-y) J(y)$$

$$(b) \quad \text{ (diagram) } = \left( \int d^4y \Delta_F(x-y) J(y) \right)^3$$

$$(c) \quad \text{ (diagram) } = (\Delta_F(x-z))^3$$

$$(d) \quad \text{ (diagram) } = \Delta_F^2(0) \Delta_F(x-z)$$

$$(e) \quad \text{ (diagram) } = \Delta_F(x-z)^2 \left( \int d^4y \Delta_F(x-y) J(y) \right) \left( \int d^4y' \Delta_F(z-y') J(y') \right)$$

$$(f) \quad \text{ (diagram) } = \Delta_F(0) \Delta_F(x-z) \left( \int d^4y \Delta_F(x-y) J(y) \right)^2$$

$$(g) \quad \text{ (diagram) } = \Delta_F(0) \Delta_F(x-z) \left( \int d^4y \Delta_F(z-y) J(y) \right)^2$$

$$(h) \quad \text{ (diagram) } = \Delta_F^2(0) \left( \int d^4y \Delta_F(x-y) J(y) \right) \left( \int d^4y' \Delta_F(z-y') J(y') \right)$$

$$(i) \quad \text{ (diagram) } = \Delta_F(x-z) \left( \int d^4y \Delta_F(x-y) J(y) \right)^2 \left( \int d^4y' \Delta_F(z-y') J(y') \right)^2$$

$$(j) \quad \text{ (diagram) } = \Delta_F(0) \left( \int d^4y \Delta_F(x-y) J(y) \right)^3 \left( \int d^4y' \Delta_F(z-y') J(y') \right)$$

$$(k) \quad \text{ (diagram) } = \Delta_F(0) \left( \int d^4y \Delta_F(x-y) J(y) \right) \left( \int d^4y' \Delta_F(z-y') J(y') \right)^3$$

$$(l) \quad \text{ (diagram) } = \left( \int d^4y \Delta_F(x-y) J(y) \right)^3 \left( \int d^4y' \Delta_F(z-y') J(y') \right)^3$$

<sup>1</sup>To create the diagrams I used the package TikZ-Feynman. If you are interested in more info, try <https://arxiv.org/abs/1601.05437>. It is good to notice, that it needs to be compiled as LuaLaTeX instead of pdfLaTeX (It saves your time and nerves).